

Yang-Mills Dynamics as Effective Nonlinear Field Theory of Composite Vector Bosons in Unified Spinorfield Models

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Unified nonlinear spinorfield models are self-regularizing quantum field theories in which all observable (elementary and non-elementary) particles are assumed to be bound states of fermionic preon fields. Due to their large masses the preons themselves are confined and below the threshold of preon production the effective dynamics of the model is only concerned with bound state reactions. In preceding papers a functional energy representation, the statistical interpretation and the dynamical equations were derived and the effective dynamics for preon-antipreon scalar boson states and three-preon fermion (and anti-fermion) states was studied in the low energy as well as in the high energy limit, leading to a functional energy representation of an effective Yukawa theory (with high energy form-factors). In this paper the effective dynamics of two-preon composite vector bosons is studied. The weak mapping of the functional energy representation of the spinorfield on to the functional energy representation for the effective vector boson dynamics (with interactions) produces a non-abelian SU(2) local gauge theory (Yang-Mills theory) for a triplet of mass-zero vector bosons in the temporal and Coulomb gauge. This special gauge is enforced by the use of the energy representation and is compatible with the nonlinear Yang-Mills dynamics (and quantization). Apart from the non-abelian Gauss-law all other field laws and constraints directly follow from the mapping procedure. The non-abelian Gauss-law is a consequence of the relativistic invariance of the effective dynamics.

PACS 11.10 Field theory

PACS 12.10 Unified field theories and models

PACS 12.35 Composite models of particles

Introduction

Unified nonlinear spinorfield (NSF) models are quantum field theories in which all observable (elementary and non-elementary) particles are assumed to be bound states of elementary fermion (preon) fields. In such models the existence and the processes of observable particles are governed by the formation and reactions of relativistic composite particles, i.e. of multi-preon states, while elementary preons are confined. Thus a relativistic composite particle quantum (field) theory is needed in order to describe the physics of observable particles. Although numerous efforts were made in the past to develop such a relativistic composite particle theory no satisfactory and systematic answers have so far been obtained in the literature for the solution of this problem. Therefore a research program has been started by the author and collaborators in this

field. In particular, a composite particle theory has been developed with respect to unified NSF models, but the results obtained there can be transferred to other types of models equally well. In this paper this investigation of composite particle physics in NSF models is continued.

NSF models are formulated by dynamical laws for selfcoupled fermion fields only. If selfcoupled fermion fields are described by NSF equations with first order derivatives (FDNSF) and local interactions the corresponding quantum field theories are non-renormalizable. To circumvent this non-renormalizability, higher order derivative nonlinear spinorfield (HDNSF) equations can be used. These equations exhibit selfregularizing properties but lead to indefinite metric in the corresponding state spaces. Thus in these models the treatment of composite particle theory is partially merged with the solution of problems related to the special model under consideration. This is the prize one has to pay for the extreme simplicity of HDNSF models connected with the assumption that only elementary fermions are the constituents of matter.

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The preceding investigations in this field are contained in papers of Grosser and Lauxmann [1], Grosser [2], Grosser, Hailer, Hornung, Lauxmann and Stumpf [3] and of the author [4, 5]. An extensive discussion of the results and of the mathematical techniques is given in [5]. After having formulated the model and obtained a first insight into the formation of bound states etc. in [1, 2, 3, 4], the most urgent problem is the investigation of effective interactions between bound states, i.e. relativistic composite particles. In particular, it has to be proved that relativistic composite particles representing observable "elementary" particles satisfy in certain approximations the corresponding gauge theories etc. which govern the reactions of these particles if they are considered to be elementary and pointlike. A first step in this direction was performed by the author [5] where it was demonstrated that the effective interactions of composite fermions and composite scalar bosons in an HDNSF model lead to a Yukawa theory in the low energy range while in the high energy range formfactor corrections appear. The mathematical techniques used for these investigations was a "weak mapping" of the HDNSF model on to a Yukawa theory. In this paper we investigate the weak mapping of an HDNSF model on to a Yang-Mills theory, i.e. we study the effective dynamics of massless vector bosons for the case of a non-abelian local $SU(2)$ gauge group. In doing so, it has to be emphasized that this local gauge group is a result of the bound state many-particle dynamics and that it is not incorporated a priori in the HDNSF model. So by the same method and with the same model it has to be expected that one can also derive an $SU(N)$ local gauge group effective dynamics if the bound states exhibit a corresponding appropriate many-particle structure. But the treatment of this problem is postponed to subsequent investigations.

The idea of generating relativistic composite particles, in particular gauge bosons by fusion of elementary relativistic fermions was inaugurated by Jordan [6] who inferred the compositeness of the photon on the basis of a statistical argument. Subsequently de Broglie [7] put forth the proposal that the photon is composed of a neutrino and an anti-neutrino. This proposal was further developed and modified by Jordan [8], Kronig [9] and other authors. Later on, Fermi and Yang [10] proposed the pion to be a relativistic composite particle

formed by a nucleon-antinucleon bound state and finally in the theory of fusion of de Broglie [11] and in the FDNSF approach of Heisenberg [12] any existing boson is assumed to be a bound state of elementary relativistic fermions or fermion fields respectively. The radical assumption of de Broglie and of Heisenberg was not generally accepted. Due to the development and the great success of gauge theories and the introduction of supersymmetry it is a widespread belief that gauge bosons are elementary objects. Nevertheless in the course of the development of high energy physics an increasing number of bosons and gauge bosons which were initially assumed to be elementary turned out to be composite objects. For instance, in quantum chromodynamics all mesons are quark-antiquark composites etc. and gluons are elementary. However, on the preon level gauge gluons are preon composites and also the electroweak gauge bosons are assumed to be composites. For a review of the extensive literature about preon models see Lyons [13]. Thus the problem of compositeness leads to the problem whether there is at all a final microscopic quantum field theory with elementary objects and whether these objects are only fermions or fermions and bosons. With respect to our model we assume the de Broglie-Heisenberg hypothesis to be valid, but as already emphasized the methods applied here can also be applied to models where elementary bosons occur.

At the beginning of the theory of composite bosons the mechanism of the formation of relativistic multi-fermion bound states was unclear and the authors argued by means of general kinematical considerations. But since the discovery of the Bethe-Salpeter equation an enormous number of papers was published which used this equation or similar equations for the study of relativistic bound states. However, in spite of this enormous effort this line of research did practically contribute nothing to the problem of effective interactions of bound states. Rather the attempts to develop a theory of effective interactions were performed along a quite different line.

The early approach to describe a composite photon was done by means of operators. A photon operator was assumed to be a product of a neutrino and an anti-neutrino operator and the authors tried to show that such bilinear neutrino operator products satisfy the kinematics etc. of a quantized

photon theory (or not), compare for instance Pryce [14]. This is the beginning of the strong mapping approach, i.e. the mapping by means of operator products as it was defined in [5]. As the formation of bound states is not only a kinematical phenomenon, such strong mappings were applied to interacting quantum fields. The first step in this direction was taken by Juvet [15]. In analogy to the neutrino theory of light he studied the strong mapping between FDNSF models and Yukawa theories by means of local spinorfield operator products and their renormalization properties ($Z_3 = 0$ equivalence theorem). Later on more sophisticated methods to perform strong mappings were introduced which, however, finally led to the same conclusions as already drawn by Juvet. The major part of these papers is concerned with the study of a strong mapping between FDNSF models and Yukawa theories and only a minor part is concerned with the study of strong mappings for other types of models or other types of composite particles, in particular gauge bosons. In [5] a review of all these papers was given. Therefore in this paper we restrict ourselves to cite only those papers which treat strong mappings in connection with gauge bosons.

Attempts to establish the photon as a collective excitation of an FDNSF model and to prove an equivalence between this FDNSF model and QED by comparison of propagators were made by Bjorken [16] and Bialynicki-Birula [17]. Lurie and Macfarlane [18] discussed the equivalence of an FDNSF model and a Yukawa theory in the chain approximation for the case of a bound boson state. They showed that [16] and [17] can be formulated and corrected within their approach and that this approach leads to Juvet's original condition. Guralnik [19] explicitly used spinorfield operator products to define boson field operators and tried to prove an equivalence between an FDNSF model and QED via the comparison of Green-functions. Without referring to an NSF model Perkins [20] constructed bilocal neutrino field operator products and demonstrated that such operators satisfy the Maxwell equations and other relevant conditions for photon operators. An essential progress in performing equivalence proofs, i.e. in clarifying strong mappings between field theories, was made by the application of functional integration techniques. Using auxiliary field path integrals which were introduced by Coleman, Jackiw and Politzer [21]

and Gross and Neveu [22], Kugo [23] and Kikkawa [24] derived by means of functional integration equivalent path integrals for FDNSF models and abelian and non-abelian gauge theories. This method was improved by Eguchi [25] and extended by several authors, Saito and Shigemoto [26], Terazawa, Chikashige and Akama [27], Konisi, Miyata, Saito and Shigemoto [28], to establish equivalences between FDNSF models and electroweak and strong fermion-boson gauge theories. The discussion of equivalences between FDNSF models and fermion-boson coupling theories has always been closely related to the problem of making non-renormalizable FDNSF models renormalizable via a map procedure. The authors who were mainly concerned with this aspect were cited in [5]. More recent papers on generation of gauge bosons from spinorfield operators and corresponding strong mappings were published by Ramon-Medrano, Pancheri-Srivastava and Srivastava [29], Amati, Barbieri, Davis and Veneziano [30], Akdeniz, Arik, Durgut, Hortacsu, Kaptanoglu and Pak [31], Akdeniz, Arik, Hortacsu and Pak [32] and Friedman and Srivastava [33]. In all these papers the path integral techniques are applied. In [30, 31, 32, 33] nonpolynomial FDNSF interactions are used. The strong mapping between gluon fields and FDNSF models was studied on the level of Lagrangians by Chiang, Chiu, Sudarshan and Tata [34] solely in terms of renormalization constants and renormalization group equations, i.e. in an improved way of Juvet's original approach. It should, however, be emphasized that the path integral approach also necessarily leads to the renormalization considerations started by Juvet.

In spite of the great effort which has been devoted to the problem of strong mapping between NSF models and other field theories the results are unsatisfactory not only in one but in many respects. Thus since the early beginning of strong mapping procedures with the neutrino theory of light there is an ever increasing number of authors who criticized this procedure on account of various drawbacks. Before summarizing our own criticism with respect to this approach we cite the literature.

Pryce [14] showed that the neutrino theory of light, i.e. the strong mapping of neutrino fields on to photon fields leads to contradictions. Broido [35] demonstrated that the photon cannot be assumed to be a bound state of fermion-antifermion pairs within

the framework of the $Z_3 = 0$ equivalence theorem, i.e. for strong mappings of FDNSF models on to Yukawa theories. Lurie [36] emphasized that the $Z_3 = 0$ condition forces the renormalized coupling constant of the corresponding Yukawa theory to be zero, i.e. the strong mapping leads to vanishing interactions. Massidda and Tirapegui [37] studied the $Z_3 = 0$ equivalence theorem for models in lower dimensions and concluded that the strong mapping procedure does not lead to strict equivalence, which can be inferred by comparison of the spectrum of the models involved. La Camera and Wataghin [38] emphasized that the $Z_3 = 0$ condition is unacceptable. In spite of preceding attempts to improve the neutrino theory of light, Strazhev [39] pointed out the inconsistency of this theory with respect to chiral properties of neutrino and photon operators. Kerler [40] (in agreement with Symanzik) investigated in detail strong mapping procedures and concluded that an equivalence between FDNSF models and Yukawa theories as claimed in literature does not exist. Rajaraman [41] demonstrated the impossibility of strong mapping equivalences by evaluating path integrals and by means of this improved techniques he arrived at inconsistency relations and a similar result as Lurie. Eguchi [42] pointed out that equivalence proofs are only possible if the FDNSF theories possess an ultraviolet-stable fixed point. However, Tamvakis and Guralnik [43] noticed that meaningful field theoretic models in four-dimensional space-time with nontrivial fixed points wait to be discovered. Konisi and Takahashi [44] tried to improve the strong mapping procedures but arrived at the conclusion that in four dimensions nontrivial ghost fields appear in the case of the $Z_3 = 0$ limit. Banks and Zaks [45] studied the possibility of strong mappings of FDNSF models by means of the path integral formalism and argued that such mappings lead to observable violation of Lorentz invariance and not to effective gauge theories. Their criticism refers to all preceding attempts in this field. Rembiesa [46] discovered that ghost poles in the $Z_3 = 0$ limit disappear only if double-charged collective bosons are admitted and neutral bosons are excluded. He further infers that the strong mapping procedure is only a way of defining an FDNSF model by means of a Yukawa theory and not a genuine map between two independently existing models. Sharatchandra [47] carefully studied the various regularization procedures

required for making renormalization well defined. In order to get meaningful results he needs derivative coupling of infinitely high order in the current-current spinorfield interaction, i.e. the common four fermion local interaction of FDNSF models cannot be mapped. Chiang, Chiu, Sudarshan and Tata [34] emphasized that the present equivalence proofs of FDNSF models and fermion-boson coupling theories by means of strong mappings need to be reexamined as they are not sufficient to ensure equivalence. A fact which they inferred from studies of mappings in the Lee-model. Nakanishi [48] criticized the bosonization of fermions and Ellwanger [49] stated that no exact method is available to demonstrate that fermion-antifermion condensation actually takes place in a given theory. He pointed out that not only one but several fixed points are required for condensation. Kerler [50] repeated his criticism as given in [40]. In a recent paper Holten [51] investigated the possibility of obtaining composite bosons in the framework of strong mappings. By taking special care of the infrared properties of various theories he concluded that no conventional scalar, spinor or vector fields lead to finite kinetic energy terms of the composite bosons in the effective (i.e. by strong mapping obtained) Lagrangians. The only candidate which might possibly allow strong mapping composite bosons might be supergravity.

In addition to this criticism further objections can be raised to strong mapping procedures, which were partly discussed in [5]. Summarizing we can formulate the essential drawbacks of strong mapping as follows:

- i) In all approaches it is necessary to introduce cut-offs or other arbitrary regularizations, infinite and (or) vanishing renormalization constants etc. By use of such procedures and conditions it is completely unclear what "being equivalent" really means.
- ii) If bosons or (and) fermions are genuine bound states of elementary fermion fields, then under suitable kinematical conditions their behavior will differ from that of elementary pointlike particles. The variety of such phenomena cannot appropriately be described by local operator products.
- iii) The composite particle theories which are assumed to be the result of strong mappings are bound to be strictly local theories, otherwise they do not exist [5]. Therefore nonlocal

operator products are in general not admitted for the map and thus no possibility exists to detect composite particle effects.

- iv) Maintaining the assumption of elementary four-fermion interactions no way is known to treat more complex bound state situations by strong mapping as for instance three-fermion or six-fermion composites or even both kinds of bound states simultaneously.
- v) The proofs of “equivalence” by means of path integrals are incomplete. If $Z(\eta, \bar{\eta})$ and $\hat{Z}(\eta, \bar{\eta}, J)$ are the generating functions of NSF-models and Yukawa theories only the “equivalence” $Z(\eta, \bar{\eta}) \equiv \hat{Z}(\eta, \bar{\eta}, 0)$ is formally obtained. The extension of the equivalence to the full \hat{Z} is an arbitrary extrapolation.
- vi) Calculations with simplified models show that no true equivalences can be obtained by strong mapping. Rather, even in very simple models the situation is more complicated and needs a careful examination of the spectrum etc.
- vii) Strong mappings between FDNSF models and coupling theories are merely definitions of nonrenormalizable theories in terms of renormalizable coupling theories but no genuine maps.
- viii) The inverse problem, i.e. the strong mappings of coupling theories on to spinor theories lead to nonlocal four-fermion interactions.

There are some attempts to extend in various ways the strong mapping procedure described so far. Kleinert [52] and Pervushin, Reinhardt and Ebert [53] proposed to use bilocal spinorfield operator products for the definition of composite bosons. Apart from the objection iii) the evaluation of their map crucially depends on the calculation of a Green function for a nonlocal Dirac equation which itself depends on the arbitrary variable unknown composite fields as potentials. Thus no general statement can be given about a map independent of dubious approximation procedures. The same objection holds with respect to the work of Furlan and Raczka [54]. The mean field expansions of Tamvakis and Guralnik [55] and Guralnik and Tamvakis [56] as well as of Bender, Cooper and Guralnik [57] suffer from the same defect. In addition, those authors obtained equivalences only for infinite coupling constants. A new line of research has been opened by the treatment of non-compact CP^N models, cf. Holten [51]. But in such models the

gauge group invariances are already contained in the original Lagrangians and the map reveals only the possibility of transforming such physically meaningless representations into the common non-abelian gauge boson representations. Such models are of minor interest for our problem under consideration.

Some of the difficulties encountered in the above approaches can be removed if HDNSF equations are used. In this case no emphasis is laid on the attempt to make nonrenormalizable NSF models via a map renormalizable. Rather one is interested to study the formation of various bound states. Dürr and Saller [58] and Saller [59] extensively studied the possibility of the deflation and inflation of the number of independent field variables in connection with the transition from coupling theories to NSF models and vice versa. They use HDNSF equations with nonlocal four-fermion interactions, the latter being equivalent to infinitely high order derivative coupling. Then by appropriate subtractions etc. they define local and nonlocal operator products as gauge boson operators and investigate symmetry breaking etc. Emphasis is laid on the mechanism of proliferation of fields and not on strong mapping itself. With respect to strong mapping these interesting papers partially suffer from the drawbacks described above and the authors concede that so far inflation, i.e. the strong mapping of HDNSF models on realistic coupling theories has only been a program.

To circumvent all these difficulties connected with strong mapping procedures in [5] a weak mapping, i.e. a mapping between the corresponding state spaces was introduced and for the case of the relation between an HDNSF model and a Yukawa theory exemplified. The development of weak mapping procedures in nonrelativistic physics was described in [5]. The main application was done in nuclear physics culminating in the cluster physics of nucleons by Wildermuth et al. [60], Schmid [61] and Kramer [62] and in nuclear quark physics by Faessler [63], Müther et al. [64]. Weak mapping in relativistic quantum field theory uses the same idea but needs a more sophisticated elaboration with genuine field theoretic methods in order to achieve any result.

With respect to weak mapping in relativistic quantum field theory the first step was done by analysing the behavior of two-fermion bound states

in relativistic quantum mechanics. For such bound states it was shown that for the center of mass motion corresponding massive vectorfield equations or Maxwell equations etc. are satisfied. In the early papers the “fusion” of fermions was considered only on the kinematical level, subsequent papers took into account binding forces. For instance de Broglie [65], de Broglie and Winter [66], Bargmann and Wigner [67], Sredniawa [68], and Sen [69] used interaction free wave equations while Bopp [70], Rosen and Singer [71] and Clapp [72] considered wave equations with interactions. The quantum field theory requires a weak mapping not for a single composite boson but for an arbitrary number of composite bosons including their interactions etc. and is thus much more complicated than the treatment of a single composite boson equation. Nevertheless, one can take advantage of these single boson equations for the evaluation of cluster equations which define the single composite particles in quantum field theory and which are the first step in the quantum field theoretic weak mapping procedure. In particular, we shall make use of the spinor algebra representation of solutions of spin 1 Bargmann-Wigner equations.

Finally, Weinberg and Witten [73] published a theorem which seemed to exclude massless charged vector bosons as being composites. However, Sudarshan [74] showed that such a far reaching consequence cannot be drawn from the theorem mentioned above.

In the following we generate massless composite vector bosons by a suitable choice of coupling constants or fermion masses and mass differences of the basic HDNSF equation, in order to enforce vanishing mass eigenvalues of such bosons in the corresponding cluster equation. Usually massless particles are generated by a symmetry breaking mechanism, cf. Dürr and Saller [58]. But these symmetry breaking mechanisms are in general nothing more than the assumption that certain vacuum expectation values do not vanish and are thus identical with a mere adaption of constants. Therefore in principle there seems to be no great difference between the two procedures. This point of view is supported by a paper of Ahrens [75] who showed that the transition to vanishing masses suffices to produce all group theoretical peculiarities of mass zero vector bosons which are required for a proper treatment of weak mapping.

In [5] the weak mapping was studied in the low energy limit as well as in the high energy limit for an HDNSF model with composite fermions and composite scalar bosons. The high energy limit is the more interesting one since in this limit formfactors appear which indicate the deviation from the local interactions of the phenomenological theory and which lead to new experimentally verifiable predictions. On the other hand, the evaluation of this limit requires a greater calculation effort. Since in this paper we are primarily and basically interested in the possibility of obtaining phenomenological non-abelian gauge theories by weak mapping procedures we do without the calculation of formfactors and restrict ourselves to the discussion of the low energy limit.

1. Fundamentals of the Model

The general unified nonlinear spinorfield model which is assumed to be the basis of the theory is defined by the field equations

$$\begin{aligned} & [(-i\gamma^\mu \partial_\mu + m_1)(-i\gamma^\nu \partial_\nu + m_2)]_{\alpha\beta} \psi_\beta(x) \\ & = g V_{\alpha\beta\gamma\delta} \psi_\beta(x) \bar{\psi}_\gamma(x) \psi_\delta(x) \end{aligned} \quad (1.1)$$

where the index α is a superindex describing spin and isospin. In the original FDNSF model of Heisenberg [12] a massless spinorfield was used which was assumed to be a Weyl-spinor-isospinor. Due to the mass terms in (1.1) the corresponding spinorfield has to be a Dirac-spinor-isospinor. In [1] and [3] the isospin was interpreted in terms of recently published preon models. But whether the analytical results of the evaluation of (1.1) have a similarity with the proposals of such purely combinatorial preon models cannot be decided at present and depends on further investigations of the model. In the following the isospin is formally treated without any specific preon interpretation.

In contrast to the nonrenormalizability of FDNSF models and the difficulties connected with this property the model (1.1) exhibits self-regularization, relativistic invariance and locality for common canonical quantization.

For the application of an energy representation equation (1.1) has to be decomposed into an equivalent set of FDNSF equations. It was proved by the

author [4] and Grosser [2] that the set of nonlinear equations $r = 1, 2$

$$\begin{aligned} & (-i \gamma^\mu \partial_\mu + m_r)_{\alpha\beta} \varphi_{\beta r}(x) \\ &= g \lambda_r \sum_{stu} V_{\alpha\beta\gamma\delta} \varphi_{\beta s}(x) \bar{\varphi}_{\gamma t}(x) \varphi_{\delta u}(x) \end{aligned} \quad (1.2)$$

is connected with (1.1) by a biunique map where this map is defined by the compatible relations

$$\begin{aligned} \psi_\alpha(x) &= \varphi_{\alpha 1}(x) + \varphi_{\alpha 2}(x), \\ \varphi_{\alpha 1}(x) &= \lambda_1 (-i \gamma^\mu \partial_\mu + m_2)_{\alpha\beta} \psi_\beta(x), \\ \varphi_{\alpha 2}(x) &= \lambda_2 (-i \gamma^\mu \partial_\mu + m_1)_{\alpha\beta} \psi_\beta(x) \end{aligned} \quad (1.3)$$

with $\lambda_r := (-1)^r (2\Delta m)^{-1}$ and $\Delta m := 1/2(m_1 - m_2)$.

The quantization of the model was performed in [3] and [4]. It turns out that the $\psi, \bar{\psi}$ anticommutator vanishes for equal times, and therefore in the interaction term of the corresponding Lagrangian all field operators completely anticommute. This is an essential property required for an unambiguous consistent definition of the vertex operator. Due to the condition of forminvariance of the corresponding Lagrangian density against Poincaré-transformations, global isospin-transformations and permutations of the field operators the vertex operator must have the general form

$$V_{\alpha\beta\gamma\delta} = \frac{1}{2} \left(\sum_h v_{\alpha\beta}^h v_{\gamma\delta}^h - \sum_h v_{\alpha\delta}^h v_{\gamma\beta}^h \right). \quad (1.4)$$

If we explicitly introduce the spinor-isospinor indexing by writing $\alpha \equiv (\alpha, A)$ and assume the unit operator to be the vertex in isospin-space then v^h reads

$$v_{\alpha\beta}^h := \hat{v}_{\alpha\beta}^h \delta_{AB}. \quad (1.5)$$

With respect to the remaining spinorial vertex part we assume the most general chiral invariant scalar and pseudoscalar coupling $h = 1, 2$ with

$$\hat{v}_{\alpha\beta}^1 := \delta_{\alpha\beta}; \quad \hat{v}_{\alpha\beta}^2 := i \gamma_{\alpha\beta}^5 \quad (1.6)$$

which was used in the symmetry breaking FDNSF model of Nambu and Jona-Lasinio [76] and which is equivalent to the vector and pseudovector coupling of Heisenberg's FDNSF equation [12] due to the Fierz theorem. Although the kinetic part of (1.2) is not chiral invariant, this kind of coupling is needed to obtain mass zero bound states in the theory as will be demonstrated in the following.

For the following investigation it is convenient to replace the adjoint spinors by the charge conjugated

spinors. Observing the properties of (1.4), (1.5) and (1.6) we obtain from (1.2) the following set of equations ($j = 1, 2$)

$$\begin{aligned} & (i \partial_\mu \gamma_{\alpha\beta}^\mu - m_j \delta_{\alpha\beta}) \varphi_{Aj\beta} \\ &= \lambda_j g \sum_h \sum_{mlk} \hat{v}_{\alpha\beta}^h \varphi_{Am\beta} (\bar{\varphi}_{Bl\alpha} \hat{v}_{\alpha\alpha'}^h \delta_{BB'} \varphi_{B'k\alpha'}), \end{aligned} \quad (1.6a)$$

$$\begin{aligned} & (-i \partial_\mu \gamma_{\alpha\beta}^{\mu T} - m_j \delta_{\alpha\beta}) \bar{\varphi}_{Aj\beta} \\ &= \lambda_j g \sum_h \sum_{mlk} \hat{v}_{\alpha\beta}^h \bar{\varphi}_{Am\beta} (\bar{\varphi}_{Bl\alpha} \hat{v}_{\alpha\alpha'}^h \delta_{BB'} \varphi_{B'k\alpha'}). \end{aligned} \quad (1.6b)$$

The charge conjugated spinor is defined by

$$\varphi_{Ajv}^c := C_{vv'}^{-1} \bar{\varphi}_{Ajv'} \quad (1.7)$$

and taking into account the properties of the γ -algebra we obtain from (1.6a) and (1.6b) the transformed equations ($j = 1, 2$)

$$\begin{aligned} & (i \partial_\mu \gamma_{\alpha\beta}^\mu - m_j \delta_{\alpha\beta}) \varphi_{Aj\beta} \\ &= -\lambda_j g \sum_h \sum_{mj k} \hat{v}_{\alpha\beta}^h \varphi_{Am\beta} (\varphi_{Bk\alpha} \hat{v}_{\alpha\alpha'}^h C_{\alpha'v} \delta_{BB'} \varphi_{B'lv}), \end{aligned} \quad (1.8a)$$

$j = 1, 2.$

$$\begin{aligned} & (i \partial_\mu \gamma_{\alpha\beta}^\mu - m_j \delta_{\alpha\beta}) \varphi_{Aj\beta}^c \\ &= -\lambda_j g \sum_h \sum_{mlk} \hat{v}_{\alpha\beta}^h \varphi_{Am\beta}^c (\varphi_{Bk\alpha} \hat{v}_{\alpha\alpha'}^h C_{\alpha'v} \delta_{BB'} \varphi_{B'lv}^c), \end{aligned} \quad (1.8b)$$

$j = 1, 2.$

Defining the superspinors

$$\begin{aligned} \varphi_{Aj\alpha 1} &:= \varphi_{Aj\alpha}, \\ \varphi_{Aj\alpha 2} &:= \varphi_{Aj\alpha}^c, \end{aligned} \quad (1.9)$$

we can combine (1.8a) and (1.8b) into one equation

$$\begin{aligned} & \sum_{Z_2} (D_{Z_1 Z_2}^\mu \partial_\mu - m_{Z_1 Z_2}) \varphi_{Z_2} \\ &= \sum_{h Z_2 Z_3 Z_4} U_{Z_1 Z_2 Z_3 Z_4}^h \varphi_{Z_2} \varphi_{Z_3} \varphi_{Z_4} \end{aligned} \quad (1.10)$$

with $Z := (\alpha, A, i, A)$ and

$$\begin{aligned} \alpha &= \text{spinor index } (\alpha = 1, 2, 3, 4), \\ A &= \text{isospinor index } (A = 1, 2), \\ i &= \text{auxiliary field index } (i = 1, 2), \\ A &= \text{superspinor index } (A = 1, 2), \end{aligned}$$

where the following definitions are used

$$\begin{aligned} D_{Z_1 Z_2}^\mu &:= i \gamma_{\alpha_1 \alpha_2}^\mu \delta_{A_1 A_2} \delta_{i_1 i_2} \delta_{A_1 A_2}, \\ m_{Z_1 Z_2} &:= m_{i_1} \delta_{\alpha_1 \alpha_2} \delta_{A_1 A_2} \delta_{i_1 i_2} \delta_{A_1 A_2}, \\ U_{Z_1 Z_2 Z_3 Z_4}^h &:= g \lambda_{i_1} \hat{v}_{\alpha_1 \alpha_2}^h \delta_{A_1 A_2} \delta_{A_1 A_2} (\hat{v}^h C)_{\alpha_3 \alpha_4} \delta_{A_3 A_4} \delta_{A_3 A_4} \delta_{A_4 A_2}. \end{aligned} \quad (1.11)$$

The quantum states of the model (1.1) or (1.10) respectively are described by state functionals $|\mathfrak{T}[j, a]\rangle$ with respect to the states $\{|a\rangle\}$ where $j \equiv j_Z(x)$ are sources with corresponding Z -indices. For concrete calculations it is necessary to introduce normal transforms by $|\mathfrak{T}\rangle = Z_0[j]|\mathfrak{F}\rangle$ and the energy representation of the spinorfield in terms of state functionals. Both procedures were discussed in detail in [3] and need not to be repeated here. The resulting functional equation reads

$$\begin{aligned} p_0 |\mathfrak{F}\rangle = & \sum_{ZZ_1Z_2} \int j_Z(x) i D_{ZZ_1}^0 [D_{Z_1Z_2}^k \partial_k - m_{Z_1Z_2}] \partial_{Z_2}(x) d^4x |\mathfrak{F}\rangle \\ & + \sum_{hZZ_1Z_2Z_3Z_4} \int j_Z(x) i D_{ZZ_1}^0 U_{Z_1Z_2Z_3Z_4}^h d_{Z_4}(x) d_{Z_3}(x) d_{Z_2}(x) d^4x |\mathfrak{F}\rangle \end{aligned} \quad (1.12)$$

with

$$d_Z(x) := \partial_Z(x) - \sum_{Z'} \int F_{ZZ'}(x, x') j_{Z'}(x') d^4x' \quad (1.13)$$

If Eq. (1.12) is projected into configuration space it allows the transition to a one-time description of the states $|\mathfrak{F}\rangle$. Formally this can be achieved by substituting $j_Z(x) \rightarrow j_Z(\mathbf{r}) \delta(t)$ into (1.12), (1.13) and $|\mathfrak{F}\rangle$ and by writing $\partial(\mathbf{r}) \equiv \partial(\mathbf{r}, 0)$. Then we can replace (1.12) by

$$\begin{aligned} p_0 |\mathfrak{F}\rangle = & \sum_{ZZ_1Z_2} \int j_Z(\mathbf{r}) i D_{ZZ_1}^0 [D_{Z_1Z_2}^k \partial_k - m_{Z_1Z_2}] \partial_{Z_2}(\mathbf{r}) d^3r |\mathfrak{F}\rangle \\ & + \sum_{hZZ_1Z_2Z_3Z_4} \int j_Z(\mathbf{r}) i D_{ZZ_1}^0 U_{Z_1Z_2Z_3Z_4}^h d_{Z_4}(\mathbf{r}) d_{Z_3}(\mathbf{r}) d_{Z_2}(\mathbf{r}) d^3r |\mathfrak{F}\rangle \end{aligned} \quad (1.14)$$

and with the abbreviation

$$K_{I_1I_2} := K_{Z_1Z_2}(\mathbf{r}_1, \mathbf{r}_2) := i \sum_Z D_{Z_1Z}^0 (D_{ZZ_2}^k \partial_k^1 - m_{Z_1Z_2}) \delta(\mathbf{r}_1 - \mathbf{r}_2), \quad (1.15)$$

$$W_{I_1I_2I_3I_4}^h := W_{Z_1Z_2Z_3Z_4}^h(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4) := i \sum_Z D_{Z_1Z}^0 U_{ZZ_2Z_3Z_4}^h \delta(\mathbf{r}_1 - \mathbf{r}_2) \delta(\mathbf{r}_1 - \mathbf{r}_3) \delta(\mathbf{r}_1 - \mathbf{r}_4), \quad (1.16)$$

Eq. (1.14) can be written in the compact form

$$p_0 |\mathfrak{F}\rangle = \sum_{I_1I_2} j_{I_1} K_{I_1I_2} \partial_{I_2} |\mathfrak{F}\rangle + \sum_{hI_1I_2I_3I_4} j_{I_1} W_{I_1I_2I_3I_4}^h d_{I_4} d_{I_3} d_{I_2} |\mathfrak{F}\rangle =: \mathcal{H} |\mathfrak{F}\rangle. \quad (1.17)$$

We assume the spinorfield interaction term to be normal-ordered. Then by evaluating this term in the corresponding functional equation (1.17) we obtain the following formula

$$\begin{aligned} d_{I_4} d_{I_3} d_{I_2} = & \partial_{I_4} \partial_{I_3} \partial_{I_2} - \sum_K F_{I_4K} j_K \partial_{I_3I_2} - \sum_K F_{I_3K} j_K \partial_{I_2} \partial_{I_4} - \sum_K F_{I_2K} j_K \partial_{I_4} \partial_{I_3} \\ & + \sum_{KK'} F_{I_4K} F_{I_3K'} j_K j_{K'} \partial_{I_2} + \sum_{KK'} F_{I_3K} F_{I_2K'} j_K j_{K'} \partial_{I_4} \\ & + \sum_{KK'} F_{I_2K} F_{I_4K'} j_K j_{K'} \partial_{I_3} - \sum_{KK'K''} F_{I_4K} F_{I_3K'} F_{I_2K''} j_K j_{K'} j_{K''} \end{aligned} \quad (1.18)$$

i.e. by the normal-ordering renormalization prescription the local terms $F_{I_2I_3} \partial_{I_4}$, $F_{I_3I_4} \partial_{I_2}$, $F_{I_4I_2} \partial_{I_3}$ (in connection with the definition of W) drop out.

2. Leading Term Approximation

Yang-Mills-fields are bosonic fields. In order to perform a weak mapping of the quantized HDNSF model on such fields we consider only boson states of the corresponding cluster state spectrum of (1.1) and study their mapping behavior. Bosonic states can arise by the aggregation of $2n$ ($n = 1, 2, \dots$) elementary fermions and (or) antifermions. There exist some indications that the realistic Yang-Mills bosons are composites of 6 fermions and (or) antifermions. For a first step in this direction it is, however, convenient to avoid such rather complicated 6-fermion states and to first discuss simple two-fermion states.

The weak mapping of a mixed spectrum of two-preon scalar boson states and of three-preon fermion states was discussed in [5]. By a detailed investigation it turned out that the cluster state representation of the functional energy operator \mathcal{H} (definition 1.17) leads to a hierarchy of interactions where only the leading terms are of interest since the higher order terms have only a negligible influence on the physical reactions. A closer inspection of this result reveals that this hierarchy of interactions is generated by the normalization properties, the energetic properties and the number of participating wavefunctions in the various interaction terms. This situation is qualitatively not changed if we consider the interactions of vector bosons instead of scalar bosons. Therefore, for the following investigation we take over the results of [5] and discuss only those terms which were proved to be the leading terms of the boson-boson interactions.

For the case that only two-fermion cluster states are taken into account the weak mapping is defined by the introduction of the boson source operators

$$b_K = \sum_{I_1 I_2} C_K^{I_1 I_2} j_{I_1} j_{I_2} \quad (2.1)$$

and by the boson representation of $|\mathfrak{F}\rangle$

$$|\mathfrak{F}\rangle = \sum_{N=1}^{\infty} \sum_{K_1 \dots K_N} c(K_1 \dots K_N) b_{K_1} \dots b_{K_N} |0\rangle \quad (2.2)$$

cf. [5], where both definitions refer to the one-time formulation. The coefficients $C_K^{I'}$ in (2.1) are the boson wavefunctions.

The representation of the functional energy states $|\mathfrak{F}\rangle$ in terms of cluster state source operators as given by (2.1) and (2.2) induces a transformation of the corresponding functional equation (1.17), i.e. the operator \mathcal{H} of this equation which initially depends on the fermion source operators is transformed into the boson source operator representation. Thus we have in our case the equality

$$\mathcal{H} \left[j, \frac{\delta}{\delta j} \right] = \hat{\mathcal{H}} \left[b, \frac{\delta}{\delta b} \right], \quad (2.3)$$

where $\hat{\mathcal{H}}$ is the boson-transform of \mathcal{H} . Such transformations were studied in detail in [5] with respect to cluster boson states as well as cluster fermion states and we can take over the formulas of [5] by merely omitting all those parts of the transformation where composite fermion variables occur. Using the same notations as in [5] we thus obtain from [5]

$$\hat{\mathcal{H}} = \sum_{k=0}^5 \mathcal{H}_{bb}^k. \quad (2.4)$$

Before discussing the various terms of (2.4) we apply to (2.4) the leading term approximation. According to [5] this approximation can be characterized by two steps:

- i) The complete set of cluster states, i.e. of scattering states as well as of bound states is reduced to the subset of bound states;
- ii) the complete set of transformed functional energy operator terms is reduced to the subset of highest magnitude terms.

While the first step is justified by energetic estimates (decoupling theorem, extremely heavy scattering state masses), the second step is justified by interaction estimates (perturbation theory, extremely small coupling constants). According to the calculations of [5] in this case the boson energy transform (2.4) approximately goes over into

$$\hat{\mathcal{H}} \approx (\mathcal{H}_{bb}^0) + (\mathcal{H}_{bb}^3) + (\mathcal{H}_{bb}^4), \quad (2.5)$$

where the brackets symbolize step i), while \mathcal{H}_{bb}^k , $k = 1, 2, 5$ are omitted due to step ii). In [5] it is argued that even (\mathcal{H}_{bb}^3) and (\mathcal{H}_{bb}^4) do not contribute to $\hat{\mathcal{H}}$ due to energy conservation. But this argument holds only for scalar bosons but not for vector bosons. Thus in the case of vector bosons all terms in (2.5) have to be fully taken into account.

The remaining terms in (2.5) read explicitly

$$(\mathcal{H}_{bb}^0) := \sum_{I_1 I_2 K} j_{I_1} K_{I_1 I_2} (\partial_{I_2} b_K) \frac{\delta}{\delta b_K} - \sum_{\substack{h I I_2 I_3 I_4 \\ I' K}} j_I W_{I I_2 I_3 I_4}^h \{F_{I_4 I'} j_{I'} (\partial_{I_3} \partial_{I_2} b_K) + F_{I_3 I'} j_{I'} (\partial_{I_2} \partial_{I_4} b_K) + F_{I_2 I'} j_{I'} (\partial_{I_4} \partial_{I_3} b_K)\} \frac{\delta}{\delta b_K}; \quad (2.6)$$

$$(\mathcal{H}_{bb}^3) := \sum_{\substack{h I I_2 I_3 I_4 \\ K K'}} j_I W_{I I_2 I_3 I_4}^h \{(\partial_{I_3} \partial_{I_2} b_K) (\partial_{I_4} b_{K'}) - (\partial_{I_4} \partial_{I_2} b_K) (\partial_{I_3} b_{K'}) + (\partial_{I_2} b_K) (\partial_{I_4} \partial_{I_3} b_{K'})\} \frac{\delta}{\delta b_{K'}} \frac{\delta}{\delta b_K}; \quad (2.7)$$

$$(\mathcal{H}_{bb}^4) := \sum_{\substack{h I I_2 I_3 I_4 \\ I' I'' K}} j_I W_{I I_2 I_3 I_4}^h \{F_{I_4 I'} F_{I_3 I''} j_{I'} j_{I''} (\partial_{I_2} b_K) + F_{I_2 I'} F_{I_4 I''} j_{I'} j_{I''} (\partial_{I_3} b_K) + F_{I_3 I'} F_{I_2 I''} j_{I'} j_{I''} (\partial_{I_4} b_K)\} \frac{\delta}{\delta b_K}. \quad (2.8)$$

For the further evaluation of these terms their symmetry properties have to be carefully taken into account.

We start with the discussion of (2.6). If (2.1) is substituted into (2.6), if the antisymmetry of the boson wavefunctions $C_K^{I'} = -C_K^{I'I}$ is used and if it is observed that any coefficient function of the product $j_I j_{I'}$ is reduced to its antisymmetric part, then by a straightforward calculation it can be shown that (2.6) yields

$$(\mathcal{H}_{bb}^0) = \sum_{\substack{I I' \\ W W' K}} j_I j_{I'} (K_{I W} \delta_{I' W'} + \delta_{I W} K_{I' W'}) C_K^{W W'} \frac{\delta}{\delta b_K} - \frac{1}{2} \sum_{\substack{h I I_2 I_3 \\ I' I'' K}} j_I j_{I'} (\hat{W}_{I I_2 I_3}^h F_{L_3 I'} - \hat{W}_{I' I_2 I_3}^h F_{L_3 I}) C_K^{I_1 I_2} \frac{\delta}{\delta b_K}, \quad (2.9)$$

where the definition

$$\hat{W}_{I I_2 I_3}^h := \{W_{I I_2 I_3}^h\}_{\text{as}(L_1 L_2 L_3)} \quad (2.10)$$

is used. According to [5] the dual relation of (2.1) reads

$$j_I j_{I'} = \sum_{K'} R_{I I'}^{K'} b_{K'} \quad (2.11)$$

and if this relation is substituted into (2.9) we eventually obtain

$$(\mathcal{H}_{bb}^0) = \sum_{\substack{I I' \\ W W' \\ K K'}} R_{I I'}^{K'} (K_{I W} \delta_{I' W'} + \delta_{I W} K_{I' W'}) C_K^{W W'} b_{K'} \frac{\delta}{\delta b_K} - \frac{1}{2} \sum_{\substack{h I I_2 I_3 \\ I' I'' \\ K K'}} R_{I I'}^{K'} (\hat{W}_{I I_2 I_3}^h F_{L_3 I'} - \hat{W}_{I' I_2 I_3}^h F_{L_3 I}) C_K^{I_1 I_2} b_{K'} \frac{\delta}{\delta b_K}, \quad (2.12)$$

where due to the approximation step i) the summation over K' and K is extended only over the bound boson states.

In a similar way the term (2.7) can be treated and we obtain

$$(\mathcal{H}_{bb}^3) = \sum_{\substack{h I I_2 I_3 \\ I' I'' \\ K K' K''}} R_{I I'}^{K''} (\hat{W}_{I I_2 I_3}^h C_K^{I_1 I'} - \hat{W}_{I' I_2 I_3}^h C_K^{I_1 I'}) C_K^{I_1 I_2} b_{K''} \frac{\delta}{\delta b_{K'}} \frac{\delta}{\delta b_K} \quad (2.13)$$

which is ready for direct evaluation.

In order to explore the meaning of (2.8) we observe that after substitution of (2.1) this term contains the direct product of four fermion source operators and thus by means of (2.11) of two boson source operators. Furthermore, it can be shown that the direct product $b_K \otimes b_{K'}$ can be expressed in terms of $b_{K''}$. From this it follows that (2.8) finally can be written in terms of $b_{K'} \frac{\delta}{\delta b_K}$, i.e. this term contributes to the selfenergy of the bosons but not to their interactions. By a rough estimate it can further be shown that this term contributes additively in the magnitude of $m^{-1/2}$ to the mass term m in (2.9). In the leading term approximation (2.8) can thus be omitted. For the sake of brevity we do not give an explicit derivation of this result here.

3. Low Energy Boson States

The weak mapping of the self-interacting preon field on a (self-interacting) composite boson field is defined by the functional relations (2.1) and (2.2). For an evaluation of this map the general explicit form of the boson wavefunctions of (2.1) is needed. According to the leading term approximation only the bound states have to be taken into account and thus only these states have to be discussed. Furthermore, for the purposes of a first investigation of this map it suffices to consider its low energy limit. In this case it is easy to derive the general explicit form of the wavefunctions under consideration. In order to perform this derivation we have to abandon the formal notation of (2.1) and to replace it by a full indexing.

The general index I of (2.1) is defined by $I := (\mathbf{r}, Z) \equiv (\mathbf{r}, \alpha, A, i, A)$. For the following investigation it is convenient to combine the isospin index A and the superspin index A into a single index \varkappa by means of the map $(A, A) \rightarrow \varkappa$, i.e. by $\{(A, A) = (1, 1)(1, 2)(2, 1)(2, 2)\} \rightarrow \{\varkappa = 1, 2, 3, 4\}$. With this notation we have $I := (\mathbf{r}, i, \alpha, \varkappa)$ and (2.1) reads with $d^3r \equiv d\mathbf{r}$ etc.

$$b \begin{pmatrix} n \\ \mathbf{k} \\ \varrho \\ j \end{pmatrix} = \sum_{\substack{r r' \\ \alpha \alpha' \\ \varkappa \varkappa'}} \int C \begin{pmatrix} \mathbf{r}, \mathbf{r}' \\ r, r' \\ \alpha, \alpha' \\ \varkappa, \varkappa' \end{pmatrix} \begin{pmatrix} n \\ \mathbf{k} \\ \varrho \\ j \end{pmatrix} j \begin{pmatrix} \mathbf{r} \\ \alpha \\ \varkappa \end{pmatrix} j \begin{pmatrix} \mathbf{r}' \\ \alpha' \\ \varkappa' \end{pmatrix} d\mathbf{r} d\mathbf{r}', \quad (3.1)$$

where we resolved the general index K into the detailed form $K := (n, \mathbf{k}, \varrho, j)$ with

- n = internal boson excitation index,
- \mathbf{k} = center of mass momentum vector,
- ϱ = spinor state index,
- j = isospin-superspinor state index.

According to [5] in the low energy limit (as well as in the high energy limit) the dependence of boson wavefunctions on the auxiliary field indices (r, r') can be decoupled from the remaining variables. Taking over this result and decomposing the wave functions into an internal part and the center of mass part, (3.1) yields

$$b \begin{pmatrix} n \\ \mathbf{k} \\ \varrho \\ j \end{pmatrix} = \sum_{\substack{r r' \\ \alpha \alpha' \\ \varkappa \varkappa'}} U^{rr'} \int e^{i\mathbf{k}(\mathbf{r}+\mathbf{r}')/2} \cdot \chi_n^{\varrho j} \begin{pmatrix} \mathbf{r} - \mathbf{r}' \\ \alpha \\ \varkappa \end{pmatrix} j \begin{pmatrix} \mathbf{r} \\ \alpha \\ \varkappa \end{pmatrix} j \begin{pmatrix} \mathbf{r}' \\ \alpha' \\ \varkappa' \end{pmatrix} d\mathbf{r} d\mathbf{r}', \quad (3.2)$$

where the matrix U is defined by, cf. [5]

$$U^{rr'} := \begin{pmatrix} U^{11} & U^{12} \\ U^{21} & U^{22} \end{pmatrix} = c \begin{pmatrix} \frac{1}{2} \eta^{-1} - \frac{1}{2} \sqrt{2} + \eta, & \frac{1}{2} \eta^{-1} - \eta \\ \frac{1}{2} \eta^{-1} - \eta, & \frac{1}{2} \eta^{-1} + \frac{1}{2} \sqrt{2} + \eta \end{pmatrix}. \quad (3.3)$$

As $U^{rr'}$ and the center of mass part of the boson wavefunctions are symmetric with respect to an interchange of their variables the antisymmetry condition for the complete wavefunctions C_K^{IJ} leads to the condition

$$\chi_n^{\varrho j} \begin{pmatrix} \mathbf{r} - \mathbf{r}' \\ \alpha \\ \varkappa \end{pmatrix} j \begin{pmatrix} \mathbf{r}' \\ \alpha' \\ \varkappa' \end{pmatrix} = - \chi_n^{\varrho j} \begin{pmatrix} \mathbf{r}' - \mathbf{r} \\ \alpha' \\ \varkappa' \end{pmatrix} j \begin{pmatrix} \mathbf{r} \\ \alpha \\ \varkappa \end{pmatrix} \quad (3.4)$$

for the internal wavefunctions χ . This condition can be satisfied in various ways.

Without loss of generality we may expand χ in terms of a Dirac algebra with respect to (α, α') as well as with respect to (\varkappa, \varkappa') . If we denote the corresponding basis elements by the sets $\{S_{\alpha\alpha'}^\sigma\} \{T_{\varkappa\varkappa'}^t\}$ we then obtain

$$\chi_n^{\varrho j} \begin{pmatrix} \mathbf{r} - \mathbf{r}' \\ \alpha \\ \varkappa \end{pmatrix} j \begin{pmatrix} \mathbf{r}' \\ \alpha' \\ \varkappa' \end{pmatrix} = \sum_{\sigma t} \chi_n^{\varrho j} \begin{pmatrix} \mathbf{r} - \mathbf{r}' \\ \alpha \\ \varkappa \end{pmatrix} j \begin{pmatrix} \mathbf{r}' \\ \alpha' \\ \varkappa' \end{pmatrix} S_{\alpha\alpha'}^\sigma T_{\varkappa\varkappa'}^t. \quad (3.5a)$$

For the following investigation it will turn out that the various basis elements themselves of the expansion (3.5a) are appropriate boson states, i.e. it is possible to identify the spinor state index q with σ and the isospinor-superspinor state index j with t . Therefore, we consider the set of internal wavefunctions

$$\chi_n^{\sigma t}(\mathbf{r} - \mathbf{r}' | \mathbf{k}) := \chi_n^{\sigma t}(\mathbf{r} - \mathbf{r}' | \mathbf{k}) S_{\alpha\alpha'}^{\sigma} T_{\kappa\kappa'}^t. \quad (3.5b)$$

In the low energy limit, i.e. for small values of \mathbf{k} the dependence of the internal wavefunctions (3.5) on \mathbf{k} can be neglected. Furthermore we assume that besides the bound boson groundstate no low energy excited bound boson states exist, so that we have to take into account only $n = 0$. This remaining groundstate must then be symmetric in the space coordinates, i.e.

$$\lim_{\mathbf{k} \rightarrow 0} \chi_0^{\sigma t}(\mathbf{r} - \mathbf{r}' | \mathbf{k}) = \chi_0^{\sigma t}(\mathbf{r} - \mathbf{r}') = \chi_0^{\sigma t}(\mathbf{r}' - \mathbf{r}). \quad (3.6)$$

Thus there remain two possibilities to satisfy (3.4): We can either require S^{σ} symmetric, T^t antisymmetric or S^{σ} antisymmetric, T^t symmetric.

The study of the interaction-free Bargmann-Wigner equations shows that the first combination has to be applied in order to obtain vector boson states. We thus try to apply this combination to the more complicated case of the fermion-fermion fusion with internal and with boundstate-boundstate interactions.

The symmetric representation of S^{σ} is defined by

$$\{S^{\sigma}\}_s := \{\gamma_{\mu} C, \Sigma_{\mu\nu} C\} \quad (3.7)$$

and the antisymmetric representation of T^t by

$$\{T^t\}_{as} := \{i\gamma_{\mu}\gamma_5 C, i\gamma_5 C, C\}, \quad (3.8)$$

where C is the charge conjugation matrix, cf. Lurie [36].

Under these assumptions in the low energy limit the relevant elements of the map (3.1) eventually read

$$b \begin{pmatrix} 0 \\ \mathbf{k} \\ \sigma \\ t \end{pmatrix} = \sum_{\substack{r r' \\ \alpha \alpha' \\ \kappa \kappa'}} U^{rr'} \int e^{i\mathbf{k}(\mathbf{r} + \mathbf{r}')/2} \cdot S_{\alpha\alpha'}^{\sigma} T_{\kappa\kappa'}^t \chi_0^{\sigma t}(\mathbf{r} - \mathbf{r}') j \begin{pmatrix} r \\ \alpha \\ \kappa \end{pmatrix} j \begin{pmatrix} r' \\ \alpha' \\ \kappa' \end{pmatrix} d\mathbf{r} d\mathbf{r}'. \quad (3.9)$$

The general explicit form of the dual relation (2.11) which is also needed for the evaluation of the transformed functional energy operator (2.5) can be derived under the same assumptions. This was extensively discussed in [5] and therefore we give only the result for the case of vector boson states. In this case (2.11) yields

$$j \begin{pmatrix} r \\ \alpha \\ \kappa \end{pmatrix} j \begin{pmatrix} r' \\ \alpha' \\ \kappa' \end{pmatrix} \approx \sum_{\sigma t} S^{rr'} \int e^{-i\mathbf{k}(\mathbf{r} + \mathbf{r}')/2} \cdot \hat{S}_{\alpha\alpha'}^{\sigma} \hat{T}_{\kappa\kappa'}^t \chi_0^{\sigma t}(\mathbf{r} - \mathbf{r}') b \begin{pmatrix} 0 \\ \mathbf{k} \\ \sigma \\ t \end{pmatrix} d\mathbf{k}, \quad (3.10)$$

where the matrix S is defined by, cf. [5]

$$S^{rr'} := \begin{pmatrix} S^{11} & S^{12} \\ S^{21} & S^{22} \end{pmatrix} = c \begin{pmatrix} \frac{1}{2} \eta^{-1} - \frac{1}{2} \sqrt{2} + \eta, & -\frac{1}{2} \eta^{-1} + \eta \\ -\frac{1}{2} \eta^{-1} + \eta, & \frac{1}{2} \eta^{-1} + \frac{1}{2} \sqrt{2} + \eta \end{pmatrix} \quad (3.11)$$

and $\{\hat{S}^{\sigma}\}$ and $\{\hat{T}^t\}$ are the sets of dual elements with respect to (3.7) and (3.8). The bound boson states which occur in the low energy mapping relation (3.9) are defined to be eigenstates of the functional cluster operator \mathcal{H}^0 which was defined in [4] and [5]. But we do not intend to give a detailed discussion of corresponding eigenstate calculations in this paper. Rather we aim at a suitable evaluation for the boson transformed operator (\mathcal{H}_{bb}^0) of \mathcal{H}^0 which is required for a detailed analysis of the map. For this (approximate) evaluation only some general properties of the internal boson state wavefunction (3.6) are needed, i.e. the general explicit form of the map (3.9) and (3.10) suffices to gain the desired insight into the mechanism of this mapping procedure. In this section we only evaluate (\mathcal{H}_{bb}^0) .

We decompose (\mathcal{H}_{bb}^0) of (2.12) into two parts

$$(\mathcal{H}_{bb}^0) = (\mathcal{H}_{bb}^0)^I + (\mathcal{H}_{bb}^0)^{II}, \quad (3.12)$$

where the first term denotes the kinetic energy while the second term denotes the interaction energy. Written with full indexing these parts read

$$\begin{aligned}
(\mathcal{H}_{bb}^0)^I := & \sum \int R \begin{pmatrix} \mathbf{r}, \mathbf{r}' \\ r, r' \\ \alpha, \alpha' \\ \varkappa, \varkappa' \end{pmatrix} \begin{vmatrix} 0 \\ \mathbf{k}' \\ \sigma' \\ t' \end{vmatrix} \left[K \begin{pmatrix} \mathbf{r}, \mathbf{s} \\ r, s \\ \alpha, \alpha' \\ \varkappa, \varkappa' \end{pmatrix} \delta \begin{pmatrix} \mathbf{r}', \mathbf{s}' \\ r', s' \\ \alpha', \alpha' \\ \varkappa', \varkappa' \end{pmatrix} + \delta \begin{pmatrix} \mathbf{r}, \mathbf{s} \\ r, s \\ \alpha, \alpha' \\ \varkappa, \varkappa' \end{pmatrix} K \begin{pmatrix} \mathbf{r}', \mathbf{s}' \\ r', s' \\ \alpha', \alpha' \\ \varkappa', \varkappa' \end{pmatrix} \right] \\
& \cdot C \begin{pmatrix} \mathbf{s}, \mathbf{s}' \\ s, s' \\ \alpha, \alpha' \\ \varkappa, \varkappa' \end{pmatrix} \begin{vmatrix} 0 \\ \mathbf{k} \\ \sigma \\ t \end{vmatrix} b \begin{pmatrix} \mathbf{k}' \\ \sigma' \\ t' \end{pmatrix} \partial \begin{pmatrix} 0 \\ \mathbf{k} \\ \sigma \\ t \end{pmatrix} dr dr' ds ds' dk dk'; \quad (3.13)
\end{aligned}$$

$$\begin{aligned}
(\mathcal{H}_{bb}^0)^{II} := & \sum \int R \begin{pmatrix} \mathbf{r}, \mathbf{r}' \\ r, r' \\ \alpha, \alpha' \\ \varkappa, \varkappa' \end{pmatrix} \begin{vmatrix} 0 \\ \mathbf{k}' \\ \sigma' \\ t' \end{vmatrix} \left[\hat{W}^h \begin{pmatrix} \mathbf{r}, \mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3 \\ r, r_1, r_2, r_3 \\ \alpha, \alpha_1, \alpha_2, \alpha_3 \\ \varkappa, \varkappa_1, \varkappa_2, \varkappa_3 \end{pmatrix} F \begin{pmatrix} \mathbf{r}_3, \mathbf{r}' \\ r_3, r' \\ \alpha_3, \alpha' \\ \varkappa_3, \varkappa' \end{pmatrix} - \hat{W}^h \begin{pmatrix} \mathbf{r}', \mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3 \\ r', r_1, r_2, r_3 \\ \alpha', \alpha_1, \alpha_2, \alpha_3 \\ \varkappa', \varkappa_1, \varkappa_2, \varkappa_3 \end{pmatrix} F \begin{pmatrix} \mathbf{r}_3, \mathbf{r} \\ r_3, r \\ \alpha_3, \alpha \\ \varkappa_3, \varkappa \end{pmatrix} \right] \\
& \cdot C \begin{pmatrix} \mathbf{r}_1, \mathbf{r}_2 \\ r_1, r_2 \\ \alpha_1, \alpha_2 \\ \varkappa_1, \varkappa_2 \end{pmatrix} \begin{vmatrix} 0 \\ \mathbf{k} \\ \sigma \\ t \end{vmatrix} b \begin{pmatrix} \mathbf{k}' \\ \sigma' \\ t' \end{pmatrix} \partial \begin{pmatrix} 0 \\ \mathbf{k} \\ \sigma \\ t \end{pmatrix} dr dr' dr_1 dr_2 dr_3 dk dk'. \quad (3.14)
\end{aligned}$$

If the combined isospinor-superspinor index \varkappa is applied to the definitions (1.11) and (1.15) and if these definitions and the wavefunctions of the map (3.9) and (3.10) are substituted into (3.13) we obtain by a straightforward calculation the expression

$$\begin{aligned}
(\mathcal{H}_{bb}^0)^I = & \sum \int e^{-ik'z} \hat{S}_{\alpha\alpha'}^{\sigma'} \chi_0^{\sigma't}(\mathbf{u}) \{ [-i \alpha_{\alpha\alpha'}^k (\frac{1}{2} \partial_{\tilde{k}} + \partial_k^\mu) + m^* \beta_{\alpha\beta}] \delta_{\alpha'\alpha'} \\
& + \delta_{\alpha\alpha'} [-i \alpha_{\alpha'\alpha'}^k (\frac{1}{2} \partial_{\tilde{k}} - \partial_k^\mu) + m^* \beta_{\alpha'\alpha'}] \} e^{ikz} S_{\alpha\alpha'}^{\sigma'} \chi_0^{\sigma t}(\mathbf{u}) b \begin{pmatrix} \mathbf{k}' \\ \sigma' \\ t \end{pmatrix} \partial \begin{pmatrix} 0 \\ \mathbf{k} \\ \sigma \\ t \end{pmatrix} dz du dk dk', \quad (3.15)
\end{aligned}$$

where $\text{tr}(US) = 1$ and $\text{tr}(\hat{T}^{t'} T^t) = \delta_{t't'}$ were used and the definitions

$$\begin{aligned}
m^* &:= \frac{1}{10} (5 - 2\sqrt{2}) (m_1 + m_2), \\
\mathbf{z} &:= \frac{1}{2} (\mathbf{r} + \mathbf{r}'), \quad \mathbf{u} := (\mathbf{r} - \mathbf{r}')
\end{aligned}$$

were introduced.

As for Lorentz-transformations the set of solutions $\{\chi_0^{\sigma t}\}$ is intermixed by the transformation laws of the set (3.7) and as due to the Lorentz-invariance of the theory the transformed set must again be a set of solutions for the same particle, it follows that the set $\{\chi_0^{\sigma t}\}$ has to be energetically degenerate and that $\chi_0^{\sigma t} = \chi_0^{\sigma't'} = \chi_0$ must hold. Thus we have

$$\langle \chi_0^{\sigma't'}, \chi_0^{\sigma t} \rangle \equiv \langle \chi_0, \chi \rangle = 1, \quad (3.16a)$$

$$\langle \chi_0^{\sigma't'}, \partial_k^\mu \chi_0^{\sigma t} \rangle \equiv \langle \chi_0, \partial_k^\mu \chi_0 \rangle = 0 \quad (3.16b)$$

for the groundstate and with the Fourier transforms

$$b_t^{\sigma'}(\mathbf{z}) := \int e^{-ik'z} b \begin{pmatrix} \mathbf{k}' \\ \sigma' \\ t \end{pmatrix} dk'; \quad \partial_t^{\sigma'}(\mathbf{z}) := \int e^{ikz} \partial \begin{pmatrix} 0 \\ \mathbf{k} \\ \sigma \\ t \end{pmatrix} dk \quad (3.17)$$

Eq. (3.15) yields

$$\begin{aligned}
(\mathcal{H}_{bb}^0)^I = & \sum_{\sigma\sigma'} \int \hat{S}_{\alpha\alpha'}^{\sigma'} b_t^{\sigma'}(\mathbf{z}) \left\{ \left[-\frac{i}{2} \alpha_{\alpha\alpha'}^k \partial_{\tilde{k}} + m^* \beta_{\alpha\alpha'} \right] \delta_{\alpha'\alpha'} \right. \\
& \left. + \delta_{\alpha\alpha'} \left[-\frac{i}{2} \alpha_{\alpha'\alpha'}^k \partial_{\tilde{k}} + m^* \beta_{\alpha'\alpha'} \right] \right\} S_{\alpha\alpha'}^{\sigma'} \partial_t^{\sigma'}(\mathbf{z}) dz. \quad (3.18)
\end{aligned}$$

The term (3.14) can be treated in a similar way. For its evaluation we write \hat{W}^h of (3.14) in the form

$$\hat{W}^h \begin{pmatrix} \mathbf{r}, \mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3 \\ r, r_1, r_2, r_3 \\ \alpha, \alpha_1, \alpha_2, \alpha_3 \\ \kappa, \kappa_1, \kappa_2, \kappa_3 \end{pmatrix} = (-1)^r \hat{V}^h \begin{pmatrix} \alpha \alpha_1 \alpha_2 \alpha_3 \\ \kappa \kappa_1 \kappa_2 \kappa_3 \end{pmatrix} \cdot \delta(\mathbf{r} - \mathbf{r}_1) \delta(\mathbf{r} - \mathbf{r}_2) \delta(\mathbf{r} - \mathbf{r}_3), \quad (3.19)$$

where \hat{V}^h follows by comparison with (1.11), (1.16) and (2.10).

Furthermore we need the explicit form of the propagator F . Due to the replacement of the adjoint spinorfields by the corresponding charge conjugated spinorfields in the basic equation (1.10) this propagator is defined by

$$F^i(\mathbf{r}, \mathbf{r}') := \delta_{AA'} \begin{pmatrix} 0 & F^i(\mathbf{r}, \mathbf{r}') \\ -F^i(\mathbf{r}', \mathbf{r}) & 0 \end{pmatrix} \quad (3.20)$$

with the free field Green-function

$$F^i(\mathbf{r}, \mathbf{r}') := \langle 0 | T \varphi_{\alpha i}^f(\mathbf{r}) \varphi_{\alpha' i}^{f\dagger}(\mathbf{r}') | 0 \rangle. \quad (3.21)$$

Under the assumptions about the wavefunctions which were made above, from (3.14) we then obtain the expression

$$\begin{aligned} (\mathcal{H}_{bb}^0)^{\text{II}} = & \eta^{-1} \sum_{h r r'} \int S^{r r'} e^{-i \mathbf{k}' \cdot \mathbf{z}} \hat{\chi}_0^{\sigma' t'}(\mathbf{u} | \mathbf{r}') \cdot [(-1)^r \hat{V}^h \begin{pmatrix} \alpha \alpha_1 \alpha_2 \alpha_3 \\ \kappa \kappa_1 \kappa_2 \kappa_3 \end{pmatrix} F^{r'}(\mathbf{u}) e^{i \mathbf{k} \mathbf{u} / 2} - (-1)^{r'} \hat{V}^h \begin{pmatrix} \alpha' \alpha_1 \alpha_2 \alpha_3 \\ \kappa' \kappa_1 \kappa_2 \kappa_3 \end{pmatrix} F^r(-\mathbf{u}) e^{-i \mathbf{k} \mathbf{u} / 2}] \\ & \cdot e^{i \mathbf{k} \cdot \mathbf{z}} \hat{\chi}_0^{\sigma t}(0 | \mathbf{k}) b \begin{pmatrix} 0 \\ \mathbf{k}' \\ \sigma' \\ t' \end{pmatrix} \partial \begin{pmatrix} 0 \\ \mathbf{k} \\ \sigma \\ t \end{pmatrix} d\mathbf{z} d\mathbf{u} d\mathbf{k} d\mathbf{k}'. \end{aligned} \quad (3.22)$$

By means of (3.11) the summations over r and r' can be explicitly evaluated and yield

$$\sum_{r r'} (-1)^r S^{r r'} F^{r'} = 2^{-1/2} (F^1 + F^2) \quad (3.23)$$

Apart from the different arguments \mathbf{u} and $-\mathbf{u}$ this result holds for both terms in (3.22). We now express the propagator (3.21) in terms of the ordinary propagators for φ^f , $\bar{\varphi}^f$ and apply the scalar approximation discussed in [5]. With $\mathbf{u} := \mathbf{r} - \mathbf{r}'$ this gives

$$\sum_{i=1}^2 F^i(\mathbf{r}, \mathbf{r}') = -\delta_{AA'} \begin{pmatrix} 0 & C_{\alpha\alpha'}^{-1} \\ C_{\alpha\alpha'}^{-1} & 0 \end{pmatrix} \eta^{-1} \delta(\mathbf{u}) = -\delta_{AA'} \sigma_{AA'}^1 C_{\alpha\alpha'}^{-1} \eta^{-1} \delta(\mathbf{u}) \equiv \eta^{-1} C_{\alpha\alpha'}^{-1} \gamma_{\kappa\kappa'}^5 \delta(\mathbf{u}) \quad (3.24)$$

and with (3.17), (3.23), and (3.24) Eq. (3.22) goes over into

$$(\mathcal{H}_{bb}^0)^{\text{II}} = \eta^{-2} \sum_h \int \hat{\chi}_0^{\sigma' t'}(0) \hat{\chi}_0^{\sigma t}(0) b_{t'}^{\sigma'}(\mathbf{z}) \partial_t^\sigma(\mathbf{z}) d\mathbf{z} [\hat{V}^h \begin{pmatrix} \alpha \alpha_1 \alpha_2 \alpha_3 \\ \kappa \kappa_1 \kappa_2 \kappa_3 \end{pmatrix} C_{\alpha\alpha'}^{-1} \gamma_{\kappa\kappa'}^5 - \hat{V}^h \begin{pmatrix} \alpha' \alpha_1 \alpha_2 \alpha_3 \\ \kappa' \kappa_1 \kappa_2 \kappa_3 \end{pmatrix} C_{\alpha\alpha'}^{-1} \gamma_{\kappa\kappa'}^5]. \quad (3.25)$$

According to [5] in the low energy approximation we have $\chi_0^{\sigma t}(0) \approx m^{3/2}$ and with $\eta := m/\Delta m$ and (3.5) we eventually obtain for (3.25)

$$(\mathcal{H}_{bb}^0)^{\text{II}} = (\Delta m)^2 m \int \hat{S}_{\alpha\alpha'}^{\sigma'} \hat{T}_{\kappa\kappa'}^{t'} M_{\alpha\alpha'}^{\sigma t} b_{t'}^{\sigma'}(\mathbf{z}) \partial_t^\sigma(\mathbf{z}) d\mathbf{z} \quad (3.26)$$

with

$$M_{\alpha\alpha'}^{\sigma t} := \sum_h [\hat{V}^h \begin{pmatrix} \alpha \alpha_1 \alpha_2 \alpha_3 \\ \kappa \kappa_1 \kappa_2 \kappa_3 \end{pmatrix} C_{\alpha\alpha'}^{-1} \gamma_{\kappa\kappa'}^5 - \hat{V}^h \begin{pmatrix} \alpha' \alpha_1 \alpha_2 \alpha_3 \\ \kappa' \kappa_1 \kappa_2 \kappa_3 \end{pmatrix} C_{\alpha\alpha'}^{-1} \gamma_{\kappa\kappa'}^5] S_{\alpha_1 \alpha_2}^\sigma T_{\kappa_1 \kappa_2}^t. \quad (3.27)$$

The expression (3.26) can be further reduced. For its explicit calculation it is convenient to consider instead of (3.27) its projection on $\hat{T}^{t'}$ given by

$$M_{\alpha\alpha'}^{\sigma}(t', t) := \hat{T}_{\kappa\kappa'}^{t'} M_{\alpha\alpha'}^{\sigma t}. \quad (3.28)$$

The calculation of (3.28) is straightforward. If one observes that the map $(A, A) \rightarrow \varkappa$ leads to a γ^5 matrix in \hat{V}^h for the isospinor-superspinor indices we have with $\hat{g} = g(2\Delta m)^{-1}$

$$\hat{V}^h \left(\begin{smallmatrix} \alpha & x_1 & x_2 & x_3 \\ \varkappa & x_1 & x_2 & x_3 \end{smallmatrix} \right) = \sum_{\substack{ij \\ \text{cyclic}}} \{(-1)^i \hat{g} (\gamma^0 \hat{v}^h)_{\alpha x_i} (\hat{v}^h C)_{x_j x_k} \delta_{\varkappa x_i} \gamma_{x_j x_k}^5\} \quad (3.29)$$

and taking into account all symmetries we obtain for (3.28)

$$M_{xx'}^\sigma(t', t) = \delta_{tt'} 2\hat{g} [-\beta_{x_0} \delta_{x'q'} - \delta_{x_0} \beta_{x'q'} + (\beta \gamma^5)_{x_0} \gamma_{x'q'}^5 + \gamma_{x_0}^5 (\beta \gamma^5)_{x'q'}] S_{q'q'}^\sigma. \quad (3.30)$$

This expression can be substituted into (3.26). The combination of (3.30), (3.26), (3.18), and (3.12) then finally leads to

$$\begin{aligned} (\mathcal{H}_{bb}^0) = \sum_{\sigma\sigma'} \int \hat{S}_{xx'}^{\sigma'} b_i^{\sigma'}(z) \left\{ \left[-\frac{i}{2} \alpha_{x_0}^k \partial_{\bar{k}} + m^* \beta_{x_0} \right] \delta_{x'q'} + g^* [-\beta_{x_0} \delta_{x'q'} + (\beta \gamma^5)_{x_0} \gamma_{x'q'}^5] \right. \\ \left. + \delta_{x_0} \left[-\frac{i}{2} \alpha_{x'q'}^k \partial_{\bar{k}} + m^* \beta_{x'q'} \right] + g^* [-\delta_{x_0} \beta_{x'q'} + \gamma_{x_0}^5 (\beta \gamma^5)_{x'q'}] \right\} S_{q'q'}^\sigma \partial_i^\sigma(z) dz \end{aligned} \quad (3.31)$$

with $g^* := 2\hat{g}(\Delta m)^2 m$. The g^* -terms are the leading term-approximation of the internal cluster interactions which produce the binding forces between the elementary preons involved in this state formation. As will be seen in Sect. 5, for an appropriate sign of \hat{g} these terms allow the construction of small or vanishing eigenmasses of the bound boson states in spite of the very large preon masses of the original spinorfield.

4. Boson-Boson Interactions

The boson-boson interaction term is given by (2.13). Written with full indexing this part reads

$$\begin{aligned} (\mathcal{H}_{bb}^3) = \sum \int R \left(\begin{smallmatrix} \mathbf{r}, \mathbf{r}' \\ r, r' \\ \alpha, \alpha' \\ \varkappa, \varkappa' \end{smallmatrix} \middle| \begin{smallmatrix} 0 \\ \mathbf{k}'' \\ \mu'' \\ j'' \end{smallmatrix} \right) \left[\hat{W}^h \left(\begin{smallmatrix} \mathbf{r}, \mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3 \\ r, r_1, r_2, r_3 \\ \alpha, \alpha_1, \alpha_2, \alpha_3 \\ \varkappa, \varkappa_1, \varkappa_2, \varkappa_3 \end{smallmatrix} \right) C \left(\begin{smallmatrix} \mathbf{r}_3, \mathbf{r}' \\ r_3, r' \\ \alpha_3, \alpha' \\ \varkappa_3, \varkappa' \end{smallmatrix} \middle| \begin{smallmatrix} 0 \\ \mathbf{k}' \\ \mu' \\ j' \end{smallmatrix} \right) - \hat{W}^h \left(\begin{smallmatrix} \mathbf{r}', \mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3 \\ r', r_1, r_2, r_3 \\ \alpha', \alpha_1, \alpha_2, \alpha_3 \\ \varkappa', \varkappa_1, \varkappa_2, \varkappa_3 \end{smallmatrix} \right) C \left(\begin{smallmatrix} \mathbf{r}_3, \mathbf{r} \\ r_3, r \\ \alpha_3, \alpha \\ \varkappa_3, \varkappa \end{smallmatrix} \middle| \begin{smallmatrix} 0 \\ \mathbf{k}' \\ \mu' \\ j' \end{smallmatrix} \right) \right] \\ \cdot C \left(\begin{smallmatrix} \mathbf{r}_1, \mathbf{r}_2 \\ r_1, r_2 \\ \alpha_1, \alpha_2 \\ \varkappa_1, \varkappa_2 \end{smallmatrix} \middle| \begin{smallmatrix} 0 \\ \mathbf{k} \\ \mu \\ j \end{smallmatrix} \right) b \left(\begin{smallmatrix} 0 \\ \mathbf{k}'' \\ \mu'' \\ j'' \end{smallmatrix} \right) \partial \left(\begin{smallmatrix} 0 \\ \mathbf{k}' \\ \mu' \\ j' \end{smallmatrix} \right) \partial \left(\begin{smallmatrix} 0 \\ \mathbf{k} \\ \mu \\ j \end{smallmatrix} \right) dr dr' dr_1 dr_2 dr_3 dk dk' dk''. \end{aligned} \quad (4.1)$$

For its evaluation we substitute the explicit form of the wavefunctions R and C which follow from (3.9) and (3.10) and the vertex expression (3.19) into (4.1). With (3.3) and (3.11) we obtain for the sums over the auxiliary field indices in (4.1)

$$\sum_{r_1 r_2} U^{r_1 r_2} = 5^{-1/2} \eta^{-1}; \quad \sum_{r r' r_3} (-1)^r S^{rr'} U^{r_3 r'} = 2 \sqrt{2} \frac{1}{5} \eta^{-1} \quad (4.2)$$

etc. If furthermore the energetic degeneracy of the wavefunctions is taken into account and the low energy approximations $\chi_0(\mathbf{u}|\mathbf{k}) \approx \chi_0(\mathbf{u}) = \chi_0(-\mathbf{u})$ and $\chi_0(0) \approx m^{3/2}$ are used, the expression (4.1) eventually goes over into

$$\begin{aligned} (\mathcal{H}_{bb}^3) = \sum_h \frac{2\sqrt{2}}{5\sqrt{5}} \eta^{-2} m^{3/2} \int \hat{S}_{\alpha\alpha'}^{\sigma'} \hat{T}_{\varkappa\varkappa'}^{t'} |\chi_0(\mathbf{u})|^2 b_{i''}^{\sigma''}(z) \partial_{i'}^{\sigma'}(z) \partial_i^{\sigma}(z) dz du \\ \cdot [\hat{V}^h \left(\begin{smallmatrix} \alpha & x_1 & x_2 & x_3 \\ \varkappa & x_1 & x_2 & x_3 \end{smallmatrix} \right) e^{i\mathbf{k}\mathbf{u}/2} S_{\alpha_3\alpha'}^{\sigma'} T_{\varkappa_3\varkappa'}^{t'} S_{\alpha_1\alpha_2}^{\sigma} T_{\varkappa_1\varkappa_2}^t - \hat{V}^h \left(\begin{smallmatrix} \alpha' & x_1 & x_2 & x_3 \\ \varkappa' & x_1 & x_2 & x_3 \end{smallmatrix} \right) e^{-i\mathbf{k}\mathbf{u}/2} S_{\alpha_3\alpha}^{\sigma'} T_{\varkappa_3\varkappa}^{t'} S_{\alpha_1\alpha_3}^{\sigma} T_{\varkappa_1\varkappa_2}^t]. \end{aligned} \quad (4.3)$$

Due to the strong concentration of χ_0 about the origin in coordinate space and due to the wavefunction normalization of χ_0 we approximately put $|\chi_0(\mathbf{u})|^2 \approx \delta(\mathbf{u})$ and obtain from (4.3) the expression

$$(\mathcal{H}_{bb}^3) = \frac{2\sqrt{2}}{5\sqrt{5}} \eta^{-2} m^{3/2} \int \hat{S}_{\alpha\alpha'}^{\sigma''} \hat{T}_{\alpha\alpha'}^{t''} b_{t''}^{\sigma''}(\mathbf{z}) \cdot \partial_{t''}^{\sigma'}(\mathbf{z}) \partial_t^{\sigma}(\mathbf{z}) d\mathbf{z} M_{\alpha\alpha'}^{\sigma' t' \sigma t} \quad (4.4)$$

with

$$M_{\alpha\alpha'}^{\sigma' t' \sigma t} := \sum_h [\hat{V}^h(\alpha_1 \alpha_2 \alpha_3) S_{\alpha_3 \alpha'}^{\sigma'} T_{\alpha_3 \alpha}^{t'} S_{\alpha_1 \alpha_2}^{\sigma} T_{\alpha_1 \alpha_2}^t - \hat{V}^h(\alpha' \alpha_1 \alpha_2 \alpha_3) S_{\alpha_3 \alpha}^{\sigma'} T_{\alpha_3 \alpha'}^{t'} S_{\alpha_1 \alpha_2}^{\sigma} T_{\alpha_1 \alpha_2}^t]. \quad (4.5)$$

The expression (4.5) can be further reduced. For its calculation we consider its projection on $\hat{S}^{\sigma''}$ and $\hat{T}^{t''}$

$$M(\alpha' \alpha' \alpha' \alpha')^{\sigma' t' \sigma t} := \hat{S}_{\alpha \alpha'}^{\sigma''} \hat{T}_{\alpha \alpha'}^{t''} M_{\alpha \alpha'}^{\sigma' t' \sigma t} \quad (4.6)$$

and substitute (3.29) and (1.5). This yields

$$M(\alpha' \alpha' \alpha' \alpha')^{\sigma' t' \sigma t} = \hat{g} \operatorname{tr} \{ T^t \gamma^5 T^{t'} (\hat{T}^{t'})^T \} \cdot \operatorname{tr} \{ [\gamma^0 S^{\sigma} C - (\gamma^0 \gamma^5) S^{\sigma} (\gamma^5 C)] S^{\sigma'} (\hat{S}^{\sigma''})^T \}. \quad (4.7)$$

In this section we explicitly calculate only the first part of (4.7).

The full set of antisymmetric T^t -states is given by (3.8). This set explicitly reads

$$\{T^t\} = \left\{ \begin{pmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{pmatrix}, \begin{pmatrix} \sigma^2 & 0 \\ 0 & -\sigma^2 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \right. \\ \left. \begin{pmatrix} 0 & \sigma^t \\ (-1)^t \sigma^t & 0 \end{pmatrix}, \quad t = 1, 2, 3 \right\}. \quad (4.8)$$

By direct calculation it can be shown that the first three elements of the set (4.8) completely decouple from the other elements with respect to the interaction (4.7). Thus the set (4.8) decomposes into two subsets which are invariant under the system dynamics and do not mix among each other. We therefore can study the dynamics of these both sets separately. As in phenomenological non-abelian SU(2) vector-boson-gauge theories triplets are the elementary representations we choose the subset

$$\{T^t\} = \left\{ \begin{pmatrix} 0 & \sigma^t \\ (-1)^t \sigma^t & 0 \end{pmatrix}, \quad t = 1, 2, 3 \right\} \quad (4.9)$$

in order to reproduce the phenomenological vector boson dynamics. For this triplet a straightforward calculation yields

$$\operatorname{tr} \{ T^t \gamma^5 T^{t'} (\hat{T}^{t'})^T \} = -2i \varepsilon^{t t' t''}, \quad (4.10)$$

where the dual set of (4.9) which is given by

$$\{\hat{T}^t\} = \left\{ \begin{pmatrix} 0 & (-1)^t \sigma^t \\ \sigma^t & 0 \end{pmatrix}, \quad t = 1, 2, 3 \right\} \quad (4.11)$$

was used. By substituting (4.6), (4.7), and (4.10) into (4.4) we finally obtain the expression

$$(\mathcal{H}_{bb}^3) = \sum_{\substack{\sigma \sigma' \sigma'' \\ t t' t''}} G^* \varepsilon^{t t' t''} \int b_{t''}^{\sigma''}(\mathbf{z}) \partial_{t''}^{\sigma'}(\mathbf{z}) \partial_t^{\sigma}(\mathbf{z}) d\mathbf{z} \quad (4.12)$$

with

$$G^* = -2i \frac{2\sqrt{2}}{5\sqrt{5}} \eta^{-2} m^{3/2} \hat{g}. \quad (4.13)$$

If the definitions of η and \hat{g} are substituted into (4.13) this formula reads

$$G^* = -i \frac{2\sqrt{2}}{5\sqrt{5}} (\Delta m) m^{-1/2} g \quad (4.14)$$

and is thus expressed in terms of the elementary constants of the model. In this intermediate step G^* is purely imaginary. In the final step i will drop out.

5. Evaluation of the Map

The weak mapping of the spinorfield model on to a vector-boson field model is defined by the boson state expansion of the spinorfield state functional (2.2) with (2.1). Due to this expansion the general energy representation has to be transformed according to (2.3) and we thus obtain from (1.17)

$$p_0 |\mathfrak{F}\rangle = \mathcal{H} \left[j, \frac{\delta}{\delta j} \right] |\mathfrak{F}\rangle = \hat{\mathcal{H}} \left[b, \frac{\delta}{\delta b} \right] |\mathfrak{F}\rangle. \quad (5.1)$$

This transformation procedure was extensively discussed in [5] and for details we refer to [5]. If the results of Sect. 2 are used, in the leading term approximation $\hat{\mathcal{H}}$ is given by

$$\hat{\mathcal{H}} = (\mathcal{H}_{bb}^0) + (\mathcal{H}_{bb}^3) \quad (5.2)$$

and the evaluation of the terms on the right-hand side of (5.2) yields for (\mathcal{H}_{bb}^0) formula (3.31) and for (\mathcal{H}_{bb}^3) formula (4.12).

Compared with the original formulas (2.6) and (2.7) for (\mathcal{H}_{bb}^0) and (\mathcal{H}_{bb}^3) , in (3.31) and (4.12) the spinorial indices remained unchanged while all

other degrees of freedom were already cast into a suitable form. Obviously for the final evaluation of the map these spinorial degrees of freedom have to be appropriately treated.

The set of spinorial basis elements is defined by (3.7). This set can be written in the form

$$\{S^\sigma\} \equiv \{s^\sigma C\} \quad (5.3)$$

with $\{s^\sigma\} := \{\gamma_\mu, i\alpha^k, (-1)^{l+1}\underline{\sigma}^l\}$ and

$$\underline{\sigma}^k := \begin{pmatrix} \sigma^k & 0 \\ 0 & \sigma^k \end{pmatrix}, \quad (5.4)$$

where for brevity we omit the bar in the following.

The decomposition of the set $\{s^\sigma\}$ into three subsets $\{\gamma_\mu\}$, $\{i\alpha^k\}$, $\{(-1)^{l+1}\sigma^l\}$ has a physical meaning. According to de Broglie [11] and Lurie [36] in

the interaction free case the set $\{\gamma_\mu\}$ corresponds to the vector potential, while $\{i\alpha^k\}$ and $\{(-1)^{l+1}\sigma^l\}$ correspond to the E -fields and B -fields respectively. We take over this interpretation for the case of a fusion model with interaction as is treated in this paper and transfer this interpretation into the functional calculus. Then the set $\{\gamma_\mu\}$ must correspond to the sources $\{b^\mu\}$ of the vector potential while $\{i\alpha^k\}$ and $\{(-1)^{l+1}\sigma^l\}$ correspond to the sources $\{b^{E_k}\}$ and $\{b^{B_l}\}$ of the electric and magnetic fields respectively. Thus (with inclusion of the gauge group indices) we may write

$$\{b_t^\sigma, \partial_t^\sigma\} \equiv \{b_t^{A_\mu}, \partial_t^{A_\mu}\} \cup \{b_t^{E_k}, \partial_t^{E_k}\} \cup \{b_t^{B_l}, \partial_t^{B_l}\}. \quad (5.5)$$

With this notation we obtain by a straightforward calculation from (3.31)

$$\begin{aligned} (\mathcal{H}_{bb}^0) = & \sum_{\sigma t \mu} \hat{S}_{\alpha\alpha'}^{\sigma'} b_t^{\sigma'}(z) \left\{ \left[\gamma^\mu, \frac{i}{2} \alpha^r \right] \partial_r + (m^* - 2g^*)[\beta, \gamma^\mu] \right\} C_{\alpha\alpha'} \partial_t^{A_\mu}(z) dz \\ & + \sum_{\sigma t k} \hat{S}_{\alpha\alpha'}^{\sigma'} b_t^{\sigma'}(z) \left\{ \left[i\alpha^k, \frac{i}{2} \alpha^r \right] \partial_r + m^*[\beta, i\alpha^k] \right\} C_{\alpha\alpha'} \partial_t^{E_k}(z) dz \\ & + \sum_{\sigma t l} \hat{S}_{\alpha\alpha'}^{\sigma'} b_t^{\sigma'}(z) \left\{ \left[(-1)^{l+1} \sigma^l, \frac{i}{2} \alpha^r \right] \partial_r + m^*[\beta, (-1)^{l+1} \sigma^l] \right\} C_{\alpha\alpha'} \partial_t^{B_l}(z) dz, \end{aligned} \quad (5.6)$$

where it has to be emphasized that this expression contains not only the kinetic energies of the preon constituents but also (in a suitable approximation) their internal cluster interaction energy. In contrast to the interaction free equations in ordinary space of de Broglie [11] and Bargmann and Wigner [67] it is just this peculiarity which allows for the transition to mass zero bosons even if the constituent preons are very heavy, while this is impossible for de Broglie and Bargmann-Wigner equations. As will be seen later the transition to mass zero bosons is accomplished by putting

$$m^* = 2g^* \quad (5.7)$$

which leads with the definitions of these quantities to the relation

$$g = (10\Delta m)^{-1} (5 - 2\sqrt{2}) \quad (5.8)$$

for the original coupling constant and the mass difference between the preon field and its accompanying ghost field. Thus by a suitable choice of the coupling constant we enforce the appearance of mass zero vector bosons.

In the following we assume (5.7) to be valid and calculate under this assumption (5.6). The straightforward calculation then gives for (5.6)

$$\begin{aligned} (\mathcal{H}_{bb}^0) = & \sum_t \int b_t^{A_0}(z) \left[i \sum_r \partial_r \partial_t^{A_r}(z) \right] dz + \sum_{ij} \int b_t^{A_i}(z) [i \partial_j \partial_t^{A_0}(z) + 2m^* i \partial_t^{E_j}(z)] dz \\ & + \sum_{\substack{tk \\ rl}} \int b_t^{E_k}(z) \left[\frac{1}{i} (-1)^l \varepsilon^{lrk} \partial_r \partial_t^{B_l}(z) \right] dz + \sum_{\substack{tl \\ kr}} \int b_t^{B_l}(z) \left[\frac{1}{i} (-1)^{l+1} \varepsilon^{krl} \partial_r \partial_t^{E_k}(z) \right] dz. \end{aligned} \quad (5.9)$$

The interaction term (4.12) can be treated in an analogous way. Applying the representation (5.3) and (5.5) to (4.12) we can reduce it to the expression

$$\begin{aligned} (\mathcal{H}_{bb}^3) = & - \sum_{\sigma'' t' l' t''} 2G^* \varepsilon^{t' t''} \int \hat{S}_{\alpha\alpha'}^{\sigma''} b_{t'}^{\sigma''}(z) \left\{ \sum_{\mu \mu'} (\gamma^0 \gamma^\mu \gamma^{\mu'})_{\alpha\nu} C_{\nu\alpha'} \partial_t^{A_{\mu'}}(z) \partial_t^{A_\mu}(z) \right. \\ & \left. + \sum_{\mu k} (\gamma^0 \gamma^\mu i\alpha^k)_{\alpha\nu} C_{\nu\alpha'} \partial_t^{E_k}(z) \partial_t^{A_\mu}(z) + \sum_{\mu l} (\gamma^0 \gamma^\mu (-1)^{l+1} \sigma^l)_{\alpha\nu} C_{\nu\alpha'} \partial_t^{B_l}(z) \partial_t^{A_\mu}(z) \right\} dz \end{aligned} \quad (5.10)$$

and further evaluation leads to the final expression

$$\begin{aligned}
 (\mathcal{H}_{bb}^3) = & - \sum_{t't''} 2G^* \varepsilon^{t't''} \int \left\{ b_t^{A_0}(\mathbf{z}) \partial_t^{A_0}(\mathbf{z}) \partial_t^{A_0}(\mathbf{z}) + \sum_{j'} b_{t'}^{A_{j'}}(\mathbf{z}) [\partial_t^{A_{j'}}(\mathbf{z}) \partial_t^{A_0}(\mathbf{z}) + \partial_t^{A_0}(\mathbf{z}) \partial_t^{A_{j'}}(\mathbf{z})] \right. \\
 & + \sum_k b_{t''}^{E_k}(\mathbf{z}) \left[\partial_t^{E_k}(\mathbf{z}) \partial_t^{A_0}(\mathbf{z}) + \sum_{jl} \varepsilon^{jkl} \partial_t^{B_l}(\mathbf{z}) (-1)^{l+1} \partial_t^{A_j}(\mathbf{z}) \right] \\
 & \left. + \sum_l b_{t''}^{B_l}(\mathbf{z}) \left[\partial_t^{B_l}(\mathbf{z}) \partial_t^{A_0}(\mathbf{z}) + \sum_{kj} \varepsilon^{jkl} \partial_t^{E_k}(\mathbf{z}) \partial_t^{A_j}(\mathbf{z}) \right] \right\} d\mathbf{z}.
 \end{aligned} \quad (5.11)$$

If we substitute $\{b_t^{B_l^\dagger}, \partial_t^{B_l^\dagger}\} = \{(-1)^{l+1} b_t^{B_l}, (-1)^{l+1} \partial_t^{B_l}\}$ into (5.9) and (5.11), the energy equation (5.1) can be written in the form

$$\begin{aligned}
 p_0 |\mathfrak{F}\rangle = & i \int \left\{ \sum_t b_t^{A_0}(\mathbf{z}) \left[\sum_r \partial_r \partial_t^{A_r}(\mathbf{z}) + \sum_{t''} C^* \varepsilon^{t''t} \partial_t^{A_0}(\mathbf{z}) \partial_t^{A_0}(\mathbf{z}) \right] \right. \\
 & + \sum_{ij} b_t^{A_j}(\mathbf{z}) \left[\partial_j \partial_t^{A_0}(\mathbf{z}) + 2m^* \partial_t^{E_j}(\mathbf{z}) + \sum_{t''} C^* \varepsilon^{t''t} (\partial_t^{A_j}(\mathbf{z}) \partial_t^{A_0}(\mathbf{z}) + \partial_t^{A_0}(\mathbf{z}) \partial_t^{A_j}(\mathbf{z})) \right] \\
 & + \sum_{tk} b_t^{E_k}(\mathbf{z}) \left[- \sum_{rl} \varepsilon^{lrk} \partial_r \partial_t^{B_l^\dagger}(\mathbf{z}) + \sum_{t''} C^* \varepsilon^{t''t} (\partial_t^{E_k}(\mathbf{z}) \partial_t^{A_0}(\mathbf{z}) + \sum_{jl} \varepsilon^{jkl} \partial_t^{B_l^\dagger}(\mathbf{z}) \partial_t^{A_j}(\mathbf{z})) \right] \\
 & \left. + \sum_{tl} b_t^{B_l^\dagger}(\mathbf{z}) \left[\sum_{kr} \varepsilon^{krl} \partial_r \partial_t^{E_k}(\mathbf{z}) + \sum_{t''} C^* \varepsilon^{t''t} (\partial_t^{B_l^\dagger}(\mathbf{z}) \partial_t^{A_0}(\mathbf{z}) + \sum_{kj} \varepsilon^{jkl} \partial_t^{E_k}(\mathbf{z}) \partial_t^{A_j}(\mathbf{z})) \right] \right\} d\mathbf{z} |\mathfrak{F}\rangle,
 \end{aligned} \quad (5.12)$$

where

$$C^* = -\frac{1}{i} 2G^* = 2 \frac{2\sqrt{2}}{5\sqrt{5}} (\Delta m) m^{-1/2} g = \frac{2\sqrt{2}}{5^2\sqrt{5}} m^{-1/2} (5 - 2\sqrt{2}) \quad (5.13)$$

is a real coupling constant. If we furthermore transform the sources $\{b_t^\sigma, \partial_t^\sigma\}$ by the affinity transformation

$$b_t^\sigma = C^* B_t^\sigma; \quad \partial_t^\sigma = (C^*)^{-1} \Delta_t^\sigma \quad (5.14)$$

which leaves the commutation relations invariant, we eventually obtain for (5.12)

$$\begin{aligned}
 p_0 |\mathfrak{F}\rangle = & i \int \left\{ \sum_t B_t^{A_0}(\mathbf{z}) \left[\sum_r \partial_r \Delta_t^{A_r}(\mathbf{z}) + \sum_{t''} \varepsilon^{t't''} \Delta_t^{A_0}(\mathbf{z}) \Delta_t^{A_0}(\mathbf{z}) \right] \right. \\
 & + \sum_{ij} B_t^{A_j}(\mathbf{z}) \left[\partial_j \Delta_t^{A_0}(\mathbf{z}) + 2m^* \Delta_t^{E_j}(\mathbf{z}) + \sum_{t''} \varepsilon^{t't''} (\Delta_t^{A_j}(\mathbf{z}) \Delta_t^{A_0}(\mathbf{z}) + \Delta_t^{A_0}(\mathbf{z}) \Delta_t^{A_j}(\mathbf{z})) \right] \\
 & + \sum_{tk} B_t^{E_k}(\mathbf{z}) \left[- \sum_{rl} \varepsilon^{lrk} \partial_r \Delta_t^{B_l^\dagger}(\mathbf{z}) + \sum_{t''} \varepsilon^{t't''} (\Delta_t^{E_k}(\mathbf{z}) \Delta_t^{A_0}(\mathbf{z}) + \sum_{jl} \varepsilon^{jkl} \Delta_t^{B_l^\dagger}(\mathbf{z}) \Delta_t^{A_j}(\mathbf{z})) \right] \\
 & \left. + \sum_{tl} B_t^{B_l^\dagger}(\mathbf{z}) \left[\sum_{rk} \varepsilon^{krl} \partial_r \Delta_t^{E_k}(\mathbf{z}) + \sum_{t''} \varepsilon^{t't''} (\Delta_t^{B_l^\dagger}(\mathbf{z}) \Delta_t^{A_0}(\mathbf{z}) + \sum_{kj} \varepsilon^{jkl} \Delta_t^{E_k}(\mathbf{z}) \Delta_t^{A_j}(\mathbf{z})) \right] \right\} d\mathbf{z} |\mathfrak{F}\rangle.
 \end{aligned} \quad (5.15)$$

For the comparison of (5.15) with the corresponding energy representation of a phenomenological vector boson gauge theory we define $2m^*$ to be the unity of mass and consider (5.15) in the temporal gauge. The temporal gauge means that the matrix elements of the field operators which are given by the coefficients of the state functional expansion (2.2) do not depend on A_0^i . Thus in the temporal gauge we have

$$\partial_t^{A_0}(\mathbf{z}) |\mathfrak{F}\rangle = (C^*)^{-1} \Delta_t^{A_0}(\mathbf{z}) |\mathfrak{F}\rangle = 0 \quad (5.15)$$

and in this gauge the energy representation (5.15) yields

$$\begin{aligned}
 p_0 |\mathfrak{F}\rangle = & i \int \left\{ \sum_{tr} B_t^{A_0}(\mathbf{z}) \partial_r \Delta_t^{A_r}(\mathbf{z}) + \sum_{rj} B_t^{A_j}(\mathbf{z}) \Delta_t^{E_j}(\mathbf{z}) \right. \\
 & + \sum_{tk} B_t^{E_k}(\mathbf{z}) \left[- \sum_{rl} \varepsilon^{lrk} \partial_r \Delta_t^{B_l^\dagger}(\mathbf{z}) + \sum_{j'l} \varepsilon^{t't''} \varepsilon^{jkl} \Delta_t^{B_l^\dagger}(\mathbf{z}) \Delta_t^{A_j}(\mathbf{z}) \right] \\
 & \left. + \sum_{tl} B_t^{B_l^\dagger}(\mathbf{z}) \left[\sum_{rk} \varepsilon^{krl} \partial_r \Delta_t^{E_k}(\mathbf{z}) + \sum_{t''} \varepsilon^{t't''} \varepsilon^{jkl} \Delta_t^{E_k}(\mathbf{z}) \Delta_t^{A_j}(\mathbf{z}) \right] \right\} d\mathbf{z} |\mathfrak{F}\rangle.
 \end{aligned} \quad (5.16)$$

Equation (5.16) can be decomposed with respect to two orthogonal subspaces. One subspace is given by the set of states $\{ \langle 0 | \Delta_{i_1}^{\sigma_1}(\mathbf{z}_1) \dots \Delta_{i_n}^{\sigma_n}(\mathbf{z}_n) \Delta_i^{A_0}(\mathbf{z}), n=0, 1, \dots, \infty, \sigma_i \neq A_0 \}$ and the other subspace by the set of states $\{ \langle 0 | \Delta_{i_1}^{\sigma_1}(\mathbf{z}_1) \dots \Delta_{i_n}^{\sigma_n}(\mathbf{z}_n), n=1, \dots, \infty, \sigma_i \neq A_0 \}$. As for the functional groundstate the relation $\langle 0 | = {}_\sigma \langle 0 | \otimes {}_0 \langle 0 |$ holds where ${}_0 \langle 0 |$ is the groundstate with respect to the temporal degrees of freedom, while ${}_\sigma \langle 0 |$ is the complementary groundstate for all other degrees of freedom, we have for $m=0, 1$, etc.

$$\begin{aligned} \langle 0 | \Delta_{i_1}^{\sigma_1}(\mathbf{z}_1) \dots \Delta_{i_n}^{\sigma_n}(\mathbf{z}_n) [\Delta_i^{A_0}(\mathbf{z})]^m \\ = {}_\sigma \langle 0 | \Delta_{i_1}^{\sigma_1}(\mathbf{z}_1) \dots \Delta_{i_n}^{\sigma_n}(\mathbf{z}_n) \otimes {}_0 \langle 0 | [\Delta_i^{A_0}(\mathbf{z})]^m \end{aligned} \quad (5.17)$$

and can thus first project equation (5.16) on the partial subspace $\{ {}_0 \langle 0 |, {}_0 \langle 0 | \Delta_i^{A_0}(\mathbf{z}) \}$. This leads to the two separate equations

$$\sum_r \partial_r \Delta_i^{A_r}(\mathbf{z}) |\mathfrak{F}\rangle = 0 \quad (5.18)$$

and

$$p_0 |\mathfrak{F}\rangle = i \sum_{ij} \int B_i^{A_j}(\mathbf{z}) \Delta_i^{E_j}(\mathbf{z}) d\mathbf{z} |\mathfrak{F}\rangle \quad (5.19)$$

$$\begin{aligned} + i \sum_{ik} \int B_i^{E_k}(\mathbf{z}) \left[- \sum_{lr} \varepsilon^{lrk} \partial_r \Delta_i^{B_l^*}(\mathbf{z}) \right. \\ \left. + \sum_{\substack{l'r'' \\ j\ l}} \varepsilon^{l'r''} \varepsilon^{jkl} \Delta_i^{B_l^*}(\mathbf{z}) \Delta_{r''}^{A_j}(\mathbf{z}) \right] d\mathbf{z} |\mathfrak{F}\rangle \\ + i \sum_{il} \int B_i^{B_l^*}(\mathbf{z}) \left[\sum_{kr} \varepsilon^{krl} \partial_r \Delta_i^{E_k}(\mathbf{z}) \right. \\ \left. + \sum_{\substack{l'r'' \\ k\ j}} \varepsilon^{l'r''} \varepsilon^{jkl} \Delta_i^{E_k}(\mathbf{z}) \Delta_{r''}^{A_j}(\mathbf{z}) \right] d\mathbf{z} |\mathfrak{F}\rangle. \end{aligned}$$

The physical meaning of these equations follows from a retranslation of the functional equation (5.18), (5.19) into ordinary operator equations. Observing the relations

$$\begin{aligned} (B_i^{A_j}(\mathbf{z}), \Delta_i^{A_j}(\mathbf{z})) &\Leftrightarrow A_j^i(\mathbf{z}), \\ (B_i^{E_k}(\mathbf{z}), \Delta_i^{E_k}(\mathbf{z})) &\Leftrightarrow E_k^i(\mathbf{z}), \\ (B_i^{B_l^*}(\mathbf{z}), \Delta_i^{B_l^*}(\mathbf{z})) &\Leftrightarrow B_l^{i*}(\mathbf{z}), \end{aligned}$$

the corresponding operator equations read

$$\sum_r \partial_r A_r^i = 0, \quad (5.20)$$

$$\partial_0 A_j^i + E_j^i = 0, \quad (5.21a)$$

$$\partial_0 E_k^i - \sum_{lr} \varepsilon^{lrk} \partial_r B_l^i + \sum_{\substack{l'r'' \\ j\ l}} \varepsilon^{l'r''} \varepsilon^{jkl} B_l^{i'} A_j^{i''} = 0, \quad (5.21b)$$

$$\partial_0 B_l^i + \sum_{kr} \varepsilon^{krl} \partial_r E_k^i + \sum_{\substack{l'r'' \\ k\ j}} \varepsilon^{l'r''} \varepsilon^{jkl} E_k^{i'} A_j^{i''} = 0. \quad (5.21c)$$

If conversely Eqs. (5.21a), (5.21b), (5.21c) are formulated in the functional energy representation we obtain (5.19), while (5.20) yields (5.18).

An equivalent formulation of (5.21a, b, c) is given by (summation convention)

$$E_i^t = -\partial_0 A_i^t, \quad (5.22a)$$

$$B_i^t = \varepsilon^{ijk} \partial_j A_k^t + \frac{1}{2} \varepsilon^{ijk} \varepsilon^{t't''} A_j^{t'} A_k^{t''}, \quad (5.22b)$$

$$\partial_0 E_i^t = \varepsilon^{ijk} \partial_j B_k^t + \varepsilon^{ijk} \varepsilon^{t't''} A_j^{t'} B_k^{t''}, \quad (5.22c)$$

and with the definition of the relativistic field tensor

$$F_{\mu\nu}^t := \begin{pmatrix} 0 & -E_1^t & -E_2^t & -E_3^t \\ E_1^t & 0 & -B_3^t & B_2^t \\ E_2^t & B_3^t & 0 & -B_1^t \\ E_3^t & -B_2^t & B_1^t & 0 \end{pmatrix} \quad (5.23)$$

we obtain for (5.22a, b, c) the set of equations

$$F_{\mu\nu}^t = (\partial_\mu A_\nu^t - \partial_\nu A_\mu^t - \varepsilon^{t't''} A_\mu^{t'} A_\nu^{t''})_{A_0=0}, \quad (5.24a)$$

$$\sum_{\mu=0}^3 \partial^\mu F_{\mu i}^t = 0; \quad i=1, 2, 3. \quad (5.24b)$$

We finally require the vector boson theory to be a relativistic invariant field theory. This enforces us to take into account $i=0$. Then the full set of equations reads

$$F_{\mu\nu}^t = (\partial_\mu A_\nu^t - \partial_\nu A_\mu^t - \varepsilon^{t't''} A_\mu^{t'} A_\nu^{t''})_{A_0=0}, \quad (5.25a)$$

$$\partial^\mu F_{\mu q}^t = 0; \quad q=0, 1, 2, 3; \quad (5.25b)$$

$$(\partial^\mu A_\mu^t)_{A_0=0} = 0. \quad (5.25c)$$

Equations (5.18), (5.19) thus are equivalent to a non-abelian SU(2) gauge vector boson theory in temporal and Coulomb gauge. It was shown by Cheng and Tsai [77] that this combined temporal-Coulomb gauge is compatible with the general dynamics and allows for a unique canonical quantization. We thus have reached our goal to demonstrate that the effective field theory for composite vector bosons in a spinorfield model leads to a non-abelian Yang-Mills field theory.

The only drawback of this method is the fact that the Gauss-law $\partial^\mu F_{\mu 0} = 0$ follows from the requirement of relativistic invariance and not by dynamical considerations. However, from the simple example of the interaction-free Bargmann-Wigner equations one learns that the Maxwellian linear Gauss-law can also be derived from the field equations if one does not insist on the use of the energy representation. Rather, in accordance with our postulate, this Gauss-law follows from a manifest relativistic

invariant map of the relativistic field equations. Such a manifest relativistic invariant map can be managed for the simple interaction-free Bargmann-Wigner equations, but it is at present beyond the scope of our experiences with maps of functional equations for fully selfinteracting spinorfields. Thus it may be hoped that further progress in this field will render superfluous the derivation of the Gauss-law by the postulate of relativistic invariance.

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